Mon. Not. R. Astron. Soc. **000**, 000–000 (2000)

Evolution of globular cluster systems in elliptical galaxies. II: power-law initial mass function

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7 October 2000

ABSTRACT
We have studied the evolution of globular cluster systems (GCS) in elliptical galaxies with a power-law initial GCS mass function (GCMF) ($f(M) \propto M^{-\alpha}$) similar to that predicted by some theoretical studies of globular clusters formation and to that of young cluster systems observed in merging galaxies.
We have carried out a survey over a large number of different host galaxies and we have considered different values for the index, α, of the initial power-law GCMF ($\alpha = 1.5, 1.8, 2.0$); we show the dependence of the main GCS final properties (mean mass and dispersion of the final GCMF, fraction of surviving clusters, radial gradient of the GCMF parameters) on the structure of the host galaxy and on the slope of the initial GCMF.
For a subsample of host galaxies with values of effective masses and radii equal to those determined using observational data for a number of giant, normal and dwarf galaxies our results show that the relation between the final GCMF properties and those of the host galaxies as well as the dependence of the final GCMF parameters on the galactocentric distance within individual galaxies differ from those observed in old GCS: the values of the final GCS mean mass are in general smaller ($4.2 \lesssim \overline{\log M_f} \lesssim 5.0$) than those observed and the galaxy-to-galaxy dispersion of $\overline{\log M_f}$ is larger than that reported by observational analyses. The results are compared with those of a companion paper in which we investigated the evolution of GCS with a log-normal initial GCMF and in which the final GCS properties were perfectly consistent with observations.

Key words: globular clusters:general – celestial mechanics, stellar dynamics – galaxies:star clusters

1 INTRODUCTION

In a recent paper (Vesperini 2000) we have investigated the evolution of the main properties of globular clusters systems (hereafter GCS) in elliptical galaxies starting with a log-normal GCS initial mass function. The choice of such a functional form for the initial mass function was motivated by the fact that all the globular cluster system mass functions (hereafter GCMF; we will indicate a globular cluster system luminosity function by GCLF) observed so far are well fitted by a log-normal distribution; in particular, a log-normal GCMF fits well the observed GCMF of clusters located in the external regions of galaxies where evolutionary processes are unlikely to have significantly altered the initial properties of clusters.

In Vesperini (2000) it was shown that the main final GCS properties resulting from the evolution of GCS with a log-normal initial GCMF and the relations between GCS properties and those of the host galaxies are in very good agreement with those reported by a number of observational analyses.

In this paper we study the evolution of GCS starting with a power-law initial GCMF. The motivation for considering this functional form for the initial GCMF comes both from some theoretical investigations of globular clusters formation (see e.g. Elmegreen & Efremov 1997, Harris & Pudritz 1994) which predict this shape for the initial GCMF and from several observational studies of young cluster systems in interacting and merging galaxies (see e.g. Schweizer et al. 1996, Miller et al.

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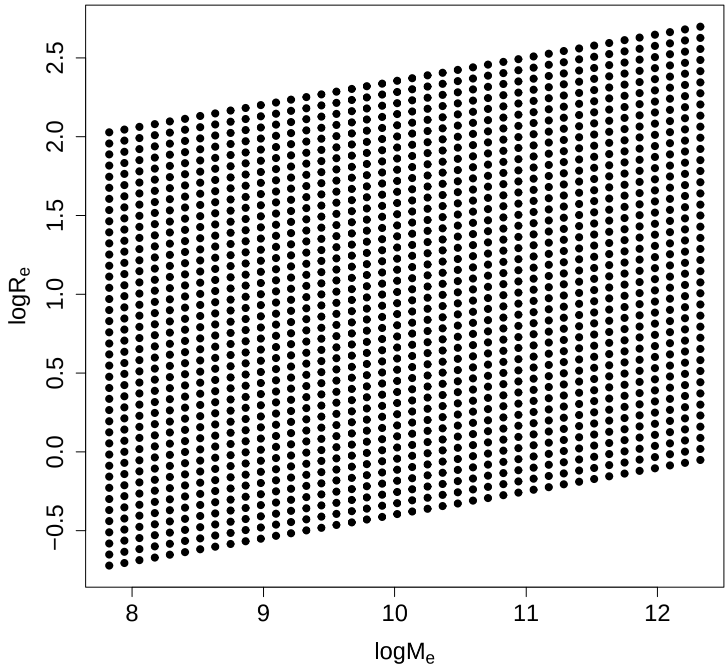


Figure 1. Set of values of the effective mass, M_e ($M_\odot$), and of the effective radius, R_e (kpc), of the host galaxies considered in the paper.

1997, Johnson et al. 1999, Zepf et al. 1999; see also Whitmore 1999 for a review and references therein) showing that these systems are characterized by a power-law luminosity function. In fact, whether the observed power-law luminosity function indeed corresponds to an underlying power-law mass function or it results from the age spread of clusters with a log-normal mass function is matter of debate (Fritze-von Alvensleben 1998, 1999, Carlson et al. 1999, Zhang & Fall 1999); moreover a recent analysis of the GCLF of young clusters in the Antennae system seems to show a turnover in the LF which would be therefore better fitted by a two-index power-law (Whitmore et al. 1999).

Most observational analyses, on the basis of the high luminosities and compact sizes observed, support the idea that the young clusters found in interacting galaxies are globular clusters which will eventually evolve into systems similar to old globular clusters (see e.g. the results of Ho & Filippenko 1996a, 1996b which strongly suggest that at least the brightest clusters must indeed be globular clusters; see also Carlson et al. 1998, 1999) but this point has been questioned too: van den Bergh (1995), on the basis of the shape of the GCLF, has claimed that these objects would be more properly classified as young open clusters rather than as young globular clusters.

Several theoretical investigations have shown that evolutionary processes can lead to the disruption of a significant number of clusters and alter the parameters and shape of the initial GCMF (Fall & Rees 1977, Fall & Malkan 1978, Caputo & Castellani 1984, Chernoff, Kochanek & Shapiro 1986, Chernoff & Shapiro 1987, Aguilar, Hut & Ostriker 1988, Vesperini, 1994, 1997, 1998, Okazaki & Tosa 1995, Capuzzo Dolcetta & Tesseri 1997, Gnedin & Ostriker 1997, Murali & Weinberg 1997a, 1997b, Ostriker & Gnedin 1997, Baumgardt 1998); in particular a power-law initial GCMF can be turned by evolutionary processes into a log-normal GCMF (or in general into a bell shaped GCMF). On the other hand it is not clear whether evolutionary processes, acting with different efficiency in galaxies with different structures, can turn an initial power-law into a log-normal GCMF with approximately universal parameters and with a weak radial variation of the GCMF parameters within individual galaxies as found in most current observational studies.

In Vesperini (2000) it has been shown that, for a log-normal initial GCMF, the galaxy-to-galaxy variation of the GCMF parameters resulting from theoretical calculations is perfectly consistent with the observed trends and that a considerable disruption of clusters does not necessarily give rise to a radial gradient of the mean mass of clusters inconsistent with observations. The goal of this paper is that of determining if the same conclusions hold for the case of GCS with a power-law initial GCMF.

The layout of the paper is the following. In §2 we briefly sketch the method used for the investigation and the initial conditions adopted; in §3 we describe the general results while in §4 we discuss in detail the implications of our results for galaxies for which observational data are available. We summarize our conclusions in §5.

2 METHOD AND INITIAL CONDITIONS

The method adopted to follow the evolution of the masses of individual globular clusters is the same used in Vesperini (2000; see also Vesperini 1998) and we refer to that paper for further details. The evolutionary processes included are mass loss due to stellar evolution, two-body relaxation, the presence of the tidal field of the host galaxy and dynamical friction; the effects of the time-variation of the tidal field for clusters on non-circular orbits (see e.g. Weinberg 1994a, 1994b, 1994c, Gnedin, Hernquist & Ostriker 1999) were not included in the N-body simulations by Vesperini & Heggie (1997) and are not considered here. The analytical expression used to determine the mass of a cluster located at distance R_g from the center of its host galaxy at time t is based on the N-body simulations carried out by Vesperini & Heggie (1997) and is given by

$$\frac{M(t)}{M_i} = 1 - \frac{\Delta M_{st.ev.}}{M_i} - \frac{0.828}{F_{CW}}t. \tag{1}$$

t is time measured in Myr, $\frac{\Delta M_{st.ev.}}{M_i}$ is the mass loss due to stellar evolution (see eq.10 in Vesperini & Heggie 1997); F_{CW} is a parameter introduced by Chernoff & Weinberg (1990), which is proportional to the initial relaxation time of the cluster and is defined as

$$F_{CW} = \frac{M_i}{M_\odot}\frac{R_g}{\mathrm{kpc}}\frac{1}{\ln N}\frac{220\mathrm{km\ s}^{-1}}{v_c}, \tag{2}$$

where N is the total initial number of stars in the cluster and v_c is the circular velocity around the host galaxy. For the host galaxy we will adopt a simple isothermal model with constant circular velocity. The effects of dynamical friction at any time t are included by removing, at that time, all clusters with time-scales of orbital decay (see e.g. Binney & Tremaine 1987) smaller than t.

The values of the effective masses, M_e, and effective radii, R_e, of the host galaxies considered are the same studied in Vesperini (2000) and they are plotted in Fig. 1.

Each GCS investigated initially contains 20000 clusters with a GCMF given by

$$dN(M) = AM^{-\alpha}dM \text{ for } 10^4\ M_\odot < M < 10^7\ M_\odot; \tag{3}$$

the values of α considered are $\alpha = 1.5,\ 1.8,\ 2.0$ which are close to those determined from observation of the GCLF of young cluster systems in merging and interacting galaxies. Clusters are initially distributed inside the host galaxy between $R_g = 0.16R_e$ and $R_g = 5R_e$ with a number of clusters per cubic kpc proportional to $R_g^{-3.5}$ which is similar to that observed for Galactic halo clusters (see also Murali & Weinberg 1997a where a similar slope for the initial radial distribution is derived for the M87 GCS).

The evolution of each GCS is followed for 15 Gyr.

3 GENERAL RESULTS

Fig. 2 shows the contour plots of the final mean mass and dispersion of the GCMF, $\overline{\log M}_f$ and σ_f, in the plane $\log M_e - \log R_e$ for the three values of α considered. The flatter the initial GCMF, the larger $\overline{\log M}_f$ is for a given model of the host galaxy. We will discuss below in section 4 the implications of our results for host galaxies with values of M_e and R_e equal to those determined by observations but we note here that the range of values spanned by $\overline{\log M}_f$ is significantly larger than that obtained in Vesperini (2000) for a log-normal initial GCMF (compare Fig.2a of Vesperini 2000 with the upper panels of Fig. 2).

Fig. 3 shows the contour plots of the fraction of the total initial number of clusters surviving after 15 Gyr, N_f/N_i, and of the ratio of the total mass of the survived clusters to the total initial mass of all clusters, $M_{GCS,f}/M_{GCS,i}$, in the plane $\log M_e - \log R_e$. For host galaxies where evaporation by internal relaxation is the dominant disruption process, the fraction of surviving clusters is smaller for steeper initial GCMF (in which the fraction of low-mass clusters is larger); on the other hand, in galaxies where dynamical friction is more important, the fraction of surviving clusters decreases for initial GCMF with smaller values of α which initially contain a larger fraction of high-mass clusters.

The left panel of Fig. 4 shows the contour plot of the difference between the final mean mass of inner clusters ($R_g < R_e$) and outer clusters ($R_g > R_e$) for $\alpha = 1.8$ (hereafter we will focus our attention on this value of α which is equal to that found in several young GCS in merging and interacting galaxies): for a large number of different host galaxies evolutionary processes lead to a significant radial gradient of the mean mass.

In general, GCS with initial GCMF containing a large fraction of low-mass clusters, such as the power-law considered here or a log-normal GCMF with a small initial value of $\overline{\log M}$ (see Vesperini 1998), are more prone to the formation of stronger radial gradients of $\overline{\log M}$ which are inconsistent with the very weak (if any) radial trends reported by observational analyses. The right panel of Fig. 4 shows that, as it happens for a log-normal initial GCMF (see Fig.4 in Vesperini 2000), a small value of $\Delta\overline{\log M}_{inn-out}$ does not necessarily imply a negligible disruption of clusters; the difference in the range of values of $\Delta\overline{\log M}_{inn-out}$ between GCS with a power-law initial GCMF considered here and those with an initial log-normal

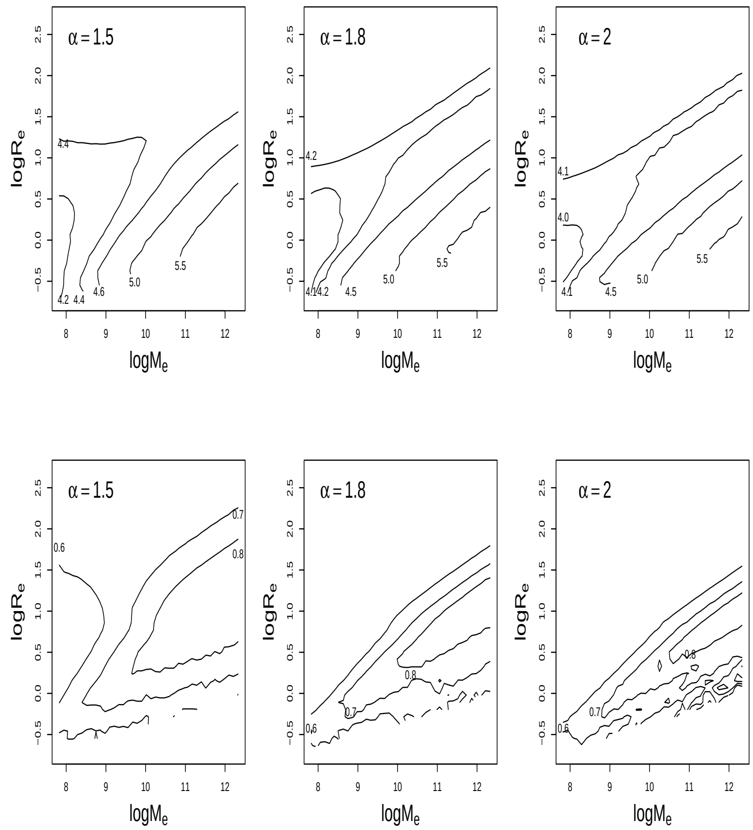


Figure 2. Contour plots of $\overline{\log M_f}$ (upper panels) and of σ_f (lower panels) in the plane $\log M_e - \log R_e$. The index of the initial power-law GCMF, α, is shown on the upper left of each panel.

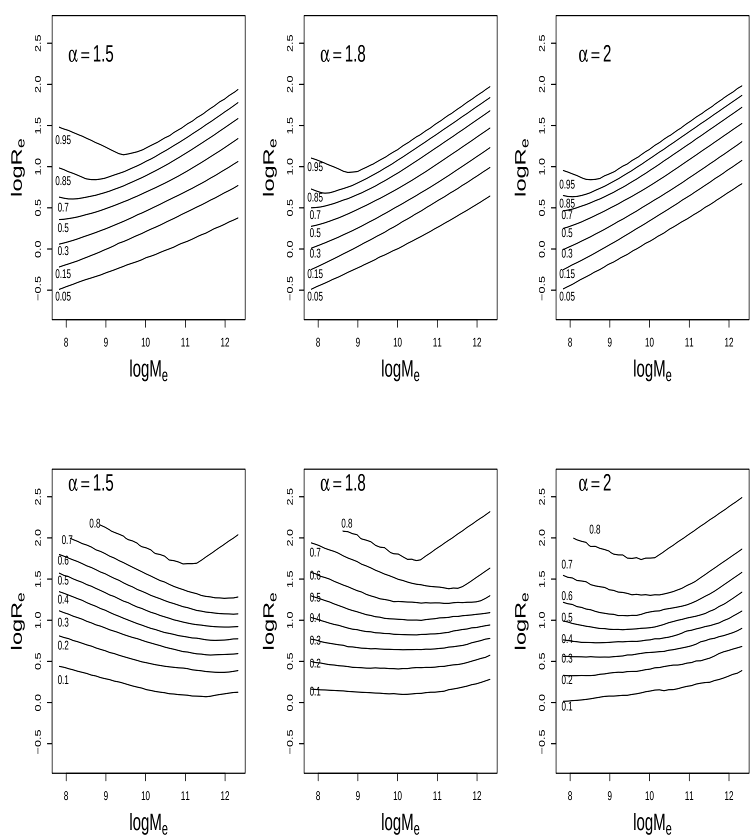


Figure 3. Contour plots of N_f/N_i (upper panels) and of $M_{GCS,f}/M_{GCS,i}$ (lower panels) in the plane $\log M_e - \log R_e$. The index of the initial power-law GCMF, α, is shown on the upper left of each panel.

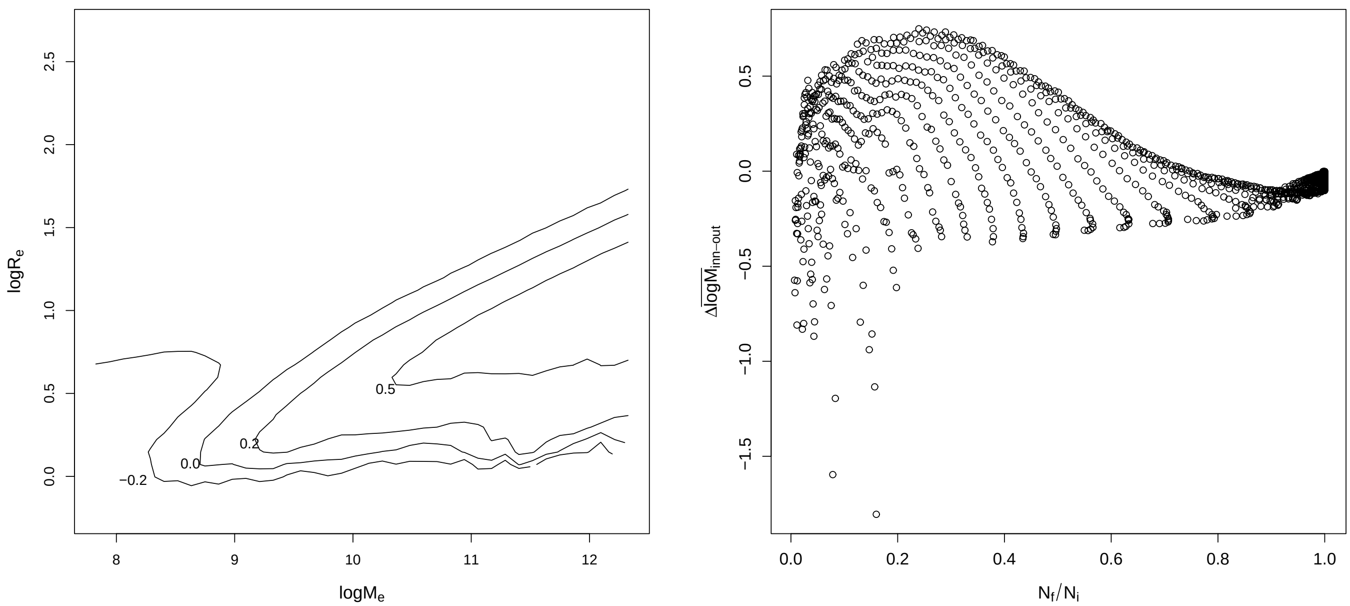


Figure 4. (Left panel) Contour plot of the difference between the final mean mass of inner ($R_g < R_e$) and outer ($R_g > R_e$) clusters, $\Delta\overline{\log M}_{inn-out}$, in the plane $\log M_e - \log R_e$. (Right panel) $\Delta\overline{\log M}_{inn-out}$ versus the fraction of surviving clusters after 15 Gyr, N_f/N_i. Both panels refer to systems with an initial power-law GCMF with $\alpha = 1.8$.

GCMF studied in Vesperini (2000) is not due to a difference in the fraction of disrupted clusters (though, in general, GCS with a power-law GCMF tend to have a smaller fraction of surviving clusters) but rather to the different mass distribution of disrupted clusters.

4 IMPLICATIONS

In this section we discuss the implications of our results for a sample of galaxies for which effective masses and radii have been determined using observational data by Burstein et al. (1997) (we adopted $H_0 = 75$ km s^{-1} Mpc^{-1}). We will focus our attention on the results obtained for $\alpha = 1.8$ but the general trends found are common to the other values of α we have studied.

4.1 Mean mass of clusters and fraction of surviving clusters

Fig. 5 shows the contour plot of $\overline{\log M}_f$ already shown in Fig. 2 and discussed in §3 with the points corresponding to the observational values of $\log M_e$ and $\log R_e$ superimposed. Figs 6a-c show $\overline{\log M}_f$, N_f/N_i, $M_{GCS,f}/M_{GCS,i}$ versus the observational values of $\log M_e$.

A recent observational analysis by Harris (2000) reports, for giant ellipticals, a mean turnover magnitude $M_V^0 = -7.33$ with a galaxy-to-galaxy rms dispersion of 0.15 (corresponding, for $M/L_V = 2$, to $\overline{\log M}_f \simeq 5.16$; the galaxy-to-galaxy rms dispersion of $\overline{\log M}_f$ is 0.06). For $10 \lesssim \log M_e \lesssim 12$, Figs 5 and 6a show instead a large spread of values of $\overline{\log M}_f$ ($4.2 < \overline{\log M}_f < 5.0$) and a general trend for $\overline{\log M}_f$ to increase as M_e decreases.

The inconsistency between the observational result of Harris (2000) and the final properties of GCS starting with an initial power-law with $\alpha = 1.8$ is further illustrated by Fig.7: this figure shows the distribution of theoretical values of $\overline{\log M}_f$ for globular cluster systems located in host galaxies with values of $\log R_e$ and $\log M_e$ equal to the observational values reported by Burstein et al. (1997) for galaxies with $\log M_e > 10.5$.

The four panels of Fig. 9 show the distribution of $\overline{\log M}_f$ for low- ($\log M_e < 9.5$), intermediate- ($9.5 < \log M_e < 10.5$) and high-mass ($\log M_e > 10.5$) ellipticals at $t = 2,\ 5,\ 10,\ 15$ Gyr; the values of $\log R_e$ and $\log M_e$ considered for these plots are those shown in Fig. 8. The region of the $\log M_e - \log R_e$ plane shown in Fig. 8 is that where most observational data fall. As shown in the observational analysis by Harris (2000) (see also Whitmore 1997), clusters in dwarf galaxies tend to have a mean mass lower than clusters in giant galaxies and a larger galaxy-to-galaxy dispersion (in dwarf galaxies, for $M/L_V = 2$, $\overline{\log M}_f = 4.99$ with a galaxy-to-galaxy dispersion equal to 0.24). It is clear from Fig.9 that, adopting a power-law initial

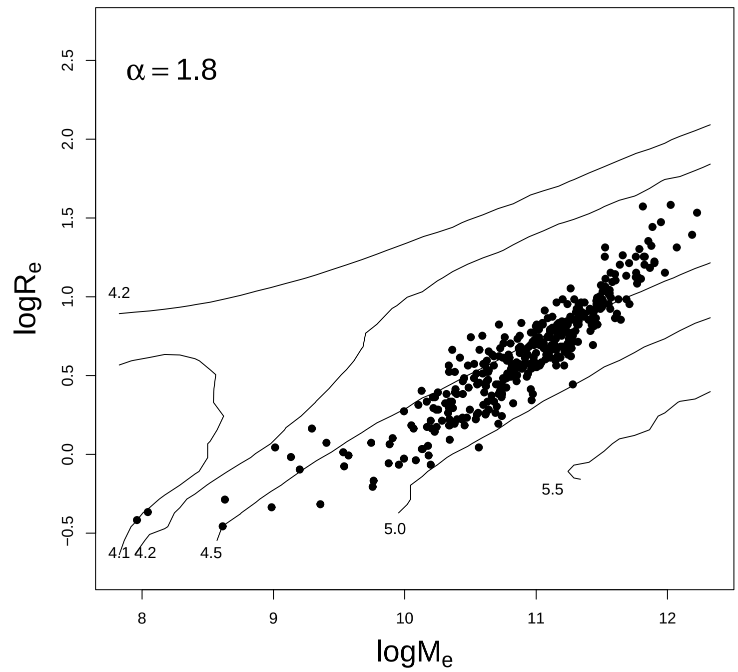


Figure 5. Contour plot of $\overline{\log M}_f$ in the plane $\log M_e - \log R_e$ (already shown in the central upper panel of Figure 2) with observational values of $\log M_e$ and $\log R_e$ for elliptical galaxies (data from Burstein et al. 1997) superimposed as filled dots. The initial GCMF adopted is a power-law function with $\alpha = 1.8$.

GCMF, the final values of $\overline{\log M}_f$, the galaxy-to-galaxy dispersion of $\overline{\log M}_f$ for different classes of galaxies, and the difference between the distribution of $\overline{\log M}_f$ for dwarf and giant ellipticals are not consistent with the current observational results. The large scatter in the distribution of $\overline{\log M}_f$ for different classes of galaxies, shown in Fig.9, is a consequence of the strong dependence of the evolution of the GCMF on the structure of the host galaxy when a power-law initial GCMF is adopted.

4.2 Radial dependence of the GCMF

For four fiducial systems with values of $\log R_e$ and $\log M_e$ shown as filled dots in Fig. 8 we have studied in larger detail the radial variation of $\overline{\log M}_f$ and of N_f/N_i.

Fig. 10 shows $\overline{\log M}_f$ and N_f/N_i vs R_g/R_e. For all the four host galaxies considered, the fraction of disrupted clusters is large also beyond R_e; only for the most massive galaxy there is no disruption beyond $R_g = 2R_e$ while for all the others disruption is significant at any galactocentric distance. As shown in the upper panel of Fig.10, $\overline{\log M}_f$ varies significantly with the galactocentric distance: the variation of $\overline{\log M}_f$ with R_g depends on the structure of the host galaxy and it is not necessarily monotonic. For the two most massive host galaxies considered, the mean mass of the innermost clusters and that of the outermost clusters differ by approximately one order of magnitude or slightly less.

Since in external galaxies only projected distances can be determined, to ease the comparison with observations, the upper panel of Fig.10 shows also the variation of $\overline{\log M}_f$ with the projected galactocentric distance. It is clear from Fig.10 that the radial gradient of $\overline{\log M}_f$ is too large to be consistent with that reported by observational studies.

It is interesting to note the contrast between the results shown in Fig. 10 and those obtained in Vesperini (2000) for GCS with a log-normal initial GCMF (see Figs 12a and 12b in Vesperini 2000): with a log-normal initial GCMF, although a significant fraction of clusters is disrupted, the radial gradient of $\overline{\log M}_f$ is, in most cases, weak and consistent with observations.

4.3 Dependence of the results on the slope of the initial GCMF

In the previous two subsections we have shown that the final properties of GCS with a power-law initial GCMF with a value of α similar to that observed in several young cluster systems are not consistent with the observational properties of old cluster

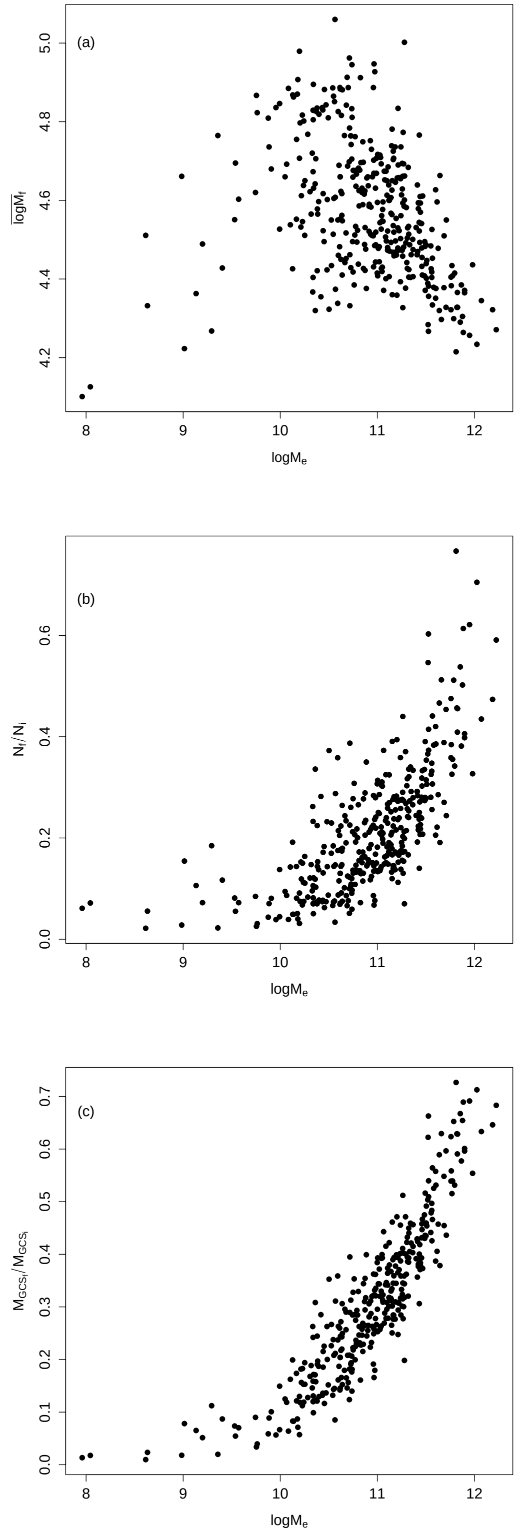


Figure 6. (a) $\overline{\log M}_f$, (b) N_f/N_i and (c) $M_{GCS,f}/M_{GCS,i}$ versus the logarithm of the effective mass of the host galaxy for a set of globular cluster systems located in host galaxies with values of R_e and M_e equal to the observational values plotted in Fig. 5. The initial GCMF adopted is a power-law function with $\alpha = 1.8$.

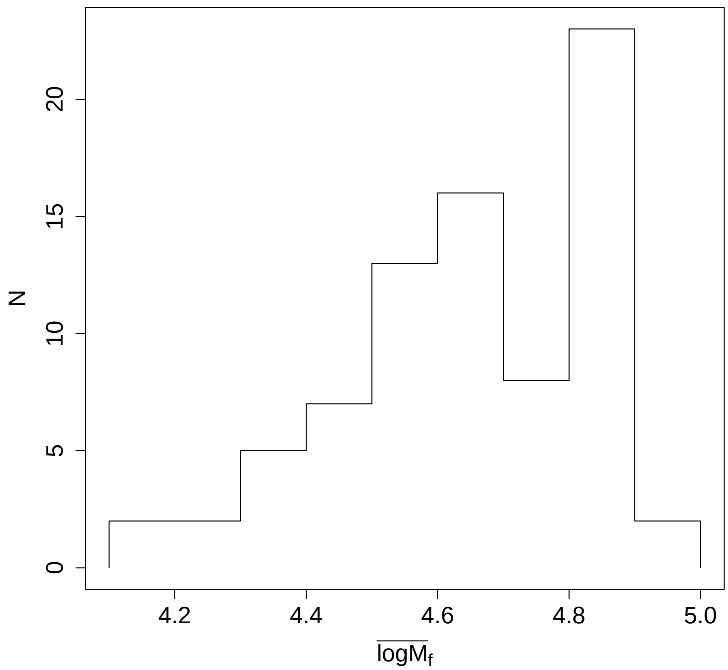


Figure 7. Distribution of $\overline{\log M}_f$ from the simulations discussed in section 4 for GCS located in host galaxies with values of $\log R_e$ and $\log M_e$ equal to the observed values (data from Burstein et al. 1997) for galaxies with $\log M_e > 10.5$. The initial power-law GCMF adopted has $\alpha = 1.8$.

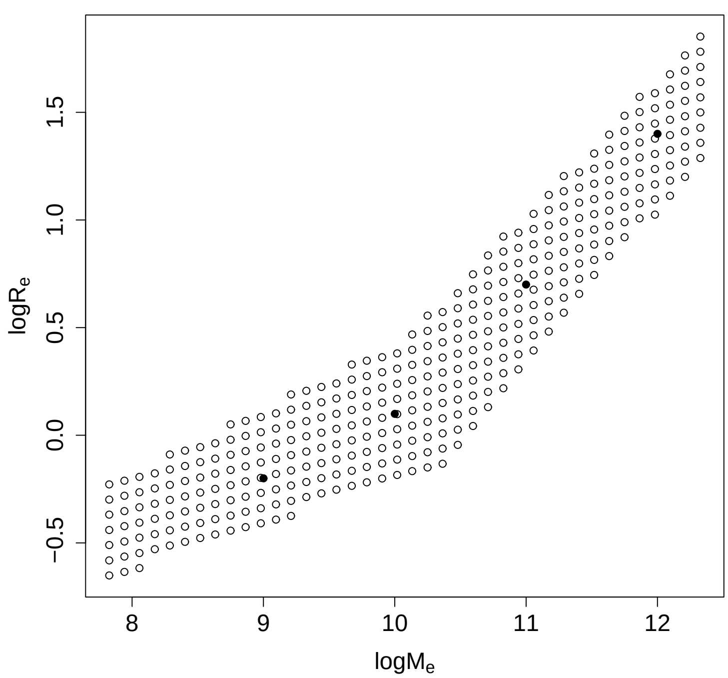


Figure 8. Values of R_e and M_e of host galaxies considered for some of the simulations discussed in section 4.1 (see Fig.9). Filled dots indicate the values of $\log M_e$ and $\log R_e$ for the host galaxies considered in §4.2 for a detailed study of the dependence of the main GCS properties on the galactocentric distance.

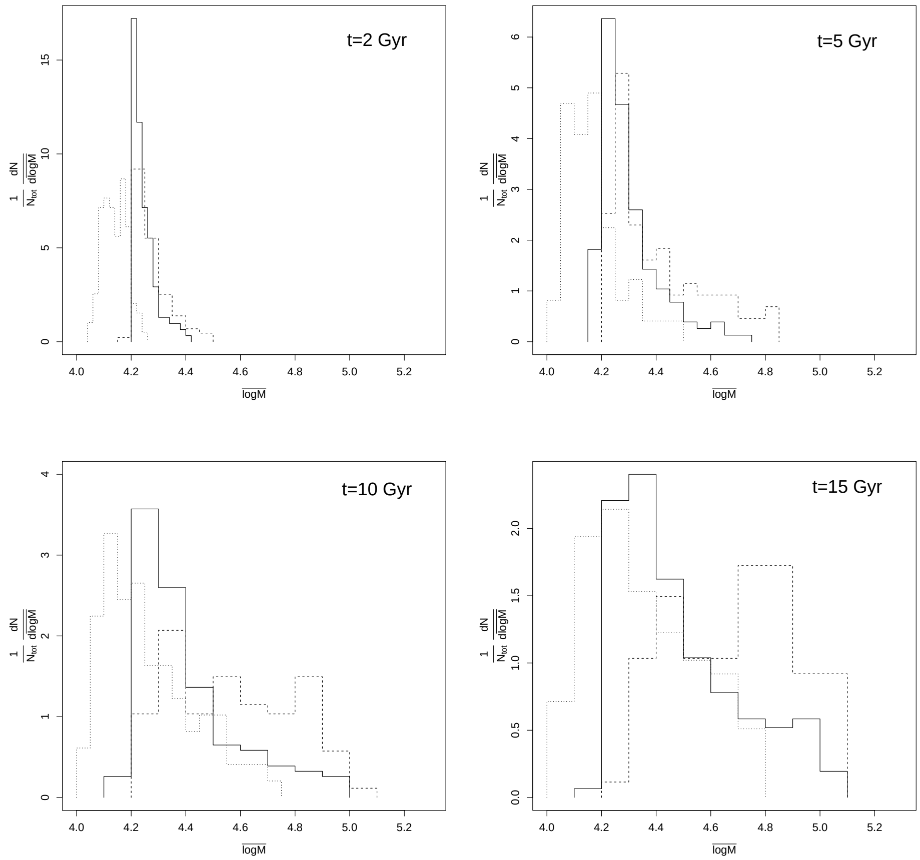


Figure 9. Evolution of the distribution of $\overline{\log M}$ of globular cluster systems located in host galaxies with values of R_e and M_e plotted in Fig. 8. In each panel, the solid line shows the distribution for globular cluster systems in host galaxies with $\log M_e > 10.5$, the dashed line that for host galaxies with $9.5 < \log M_e < 10.5$ and the dotted line that for host galaxies with $\log M_e < 9.5$.

systems. In particular the values of $\overline{\log M}_f$ for high-mass host galaxies have been found to be smaller than those observed and the galaxy-to-galaxy dispersion much larger than that observed.

Although values of α too different from that adopted above have not received any support either by theoretical studies of clusters formation or by observational analyses of young cluster systems, it is interesting to study the dependence of our results on α and explore the evolution of GCS with initial values of α outside the range considered in the previous sections.

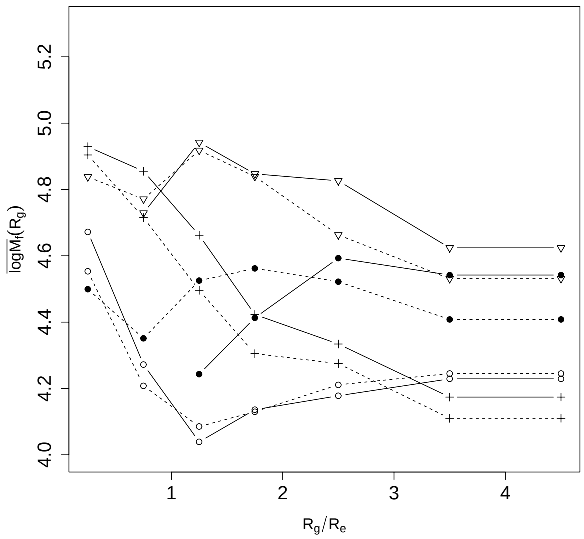
We focus our attention on high-mass galaxies with $\log M_e > 10.5$ and consider the following values of $(\log M_e,\ \log R_e)$:(12,1.4), (11.75,1.18), (11.5,1), (11.25,0.8), (11.0,0.64), (10.75,0.52), (10.5,0.4); these values span the entire strip of the $\log M_e - \log R_e$ plane covered by real galaxies with $\log M_e > 10.5$. Figs 11a-c show $\overline{\log M}_f$, N_f/N_i and $M_{GCS,f}/M_{GCS,i}$ versus α. As expected, the smaller α, the larger the fraction of surviving clusters (but note that the disruption is never negligible) and the larger the final values of $\overline{\log M}_f$. Values of $\overline{\log M}_f$ consistent with those observed can result only from initial GCMF with values of α ($\alpha \simeq 1.2 - 1.3$) significantly flatter than those observed in young cluster systems.

5 SUMMARY AND CONCLUSIONS

We have investigated the evolution of the main properties of GCS in elliptical galaxies starting with an initial power-law GCMF similar to that of young cluster systems observed in merging galaxies.

We have considered a large set of different host galaxies and we have shown the dependence of the final parameters of the GCMF, of the fraction of surviving clusters and of the ratio of the final total mass in clusters to the total initial mass of clusters on the effective radius and effective mass of the host galaxy (see Figs 2, 3, and 4). Three different values (close to those determined observationally from the GCLF of young cluster systems) of the index of the initial power-law GCMF have been explored ($\alpha = 1.5,\ 1.8,\ 2.0$).

While evolutionary processes can easily turn a GCMF with an initial power-law shape into a GCMF with a log-normal shape like that observed in old cluster systems, our investigation reveals several inconsistencies between the observed properties



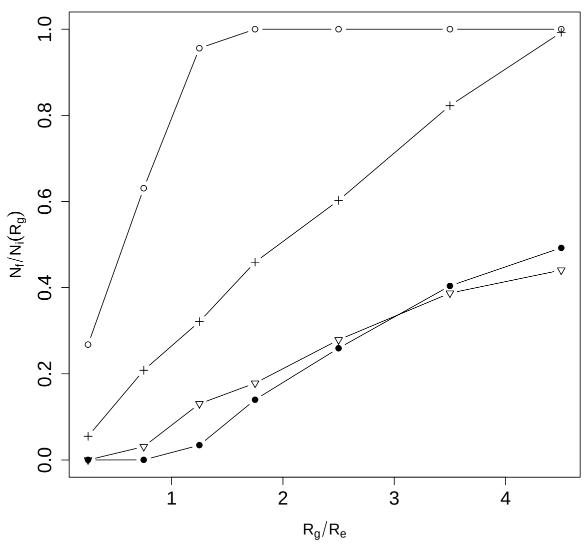


Figure 10. $\overline{\log M_f}$ (upper panel) and N_f/N_i (lower panel) versus the galactocentric distance (normalized by the effective radius of the host galaxy) for globular cluster systems in host galaxies with effective masses and radii plotted in Fig. 8 as filled dots (open dots are for $(\log M_e, \log R_e) = (12, 1.4)$, crosses for $(\log M_e, \log R_e) = (11, 0.7)$, triangles for $(\log M_e, \log R_e) = (10, 0.1)$ and filled dots for $(\log M_e, \log R_e) = (9, -0.2)$). The results are from simulations with an initial power-law GCMF with $\alpha = 1.8$. The dashed lines in the upper panel show $\overline{\log M_f}$ versus the projected galactocentric distance (normalized by the effective radius of the host galaxy).

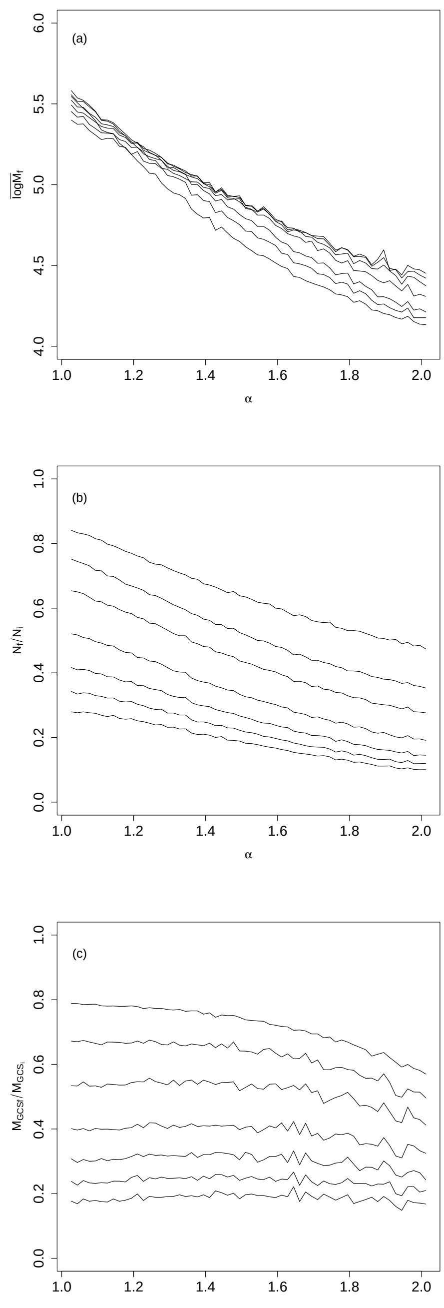


Figure 11. (a) $\overline{\log M}_f$, (b) N_f/N_i, (c) $M_{GCS,f}/M_{GCS,i}$ versus the index, α, of the initial power-law GCMF for the seven host galaxies considered in section 4.3. In panels (b) and (c), the seven curves shown correspond, from the upper to the lower one, to host galaxies with decreasing values of M_e; in panel (a) the curves shown correspond, from the upper to the lower according to their position on the right side of the panel, to host galaxies with increasing values of M_e.

of old GCS in elliptical galaxies and the theoretical results obtained adopting a power-law initial GCMF. In particular, studying the evolution of GCS in a sample of giant, normal and dwarf elliptical galaxies with effective radii and effective masses equal to those determined observationally, we have shown that:

- the theoretical values of $\overline{\log M_f}$ are in general smaller than those observed;
- the range spanned by $\overline{\log M_f}$ is significantly larger than that reported by observational studies; in particular, for galaxies with $\log M_e > 10.5$, we have shown that starting with a power-law initial GCMF with $\alpha = 1.8$, $\overline{\log M_f}$ ranges from 4.2 to 5.0 whereas observations show an approximately universal value of $\overline{\log M_f}$ ($\overline{\log M_f} \simeq 5.16$ for $M/L_V = 2$) with a very small galaxy-to-galaxy dispersion (~ 0.06);
- $\overline{\log M_f}$ varies significantly with the effective mass of the host galaxy; specifically $\overline{\log M_f}$ tends to increase with decreasing values of $\log M_e$ for $\log M_e > 10$, it reaches a peak and then decreases again for dwarf galaxies; this does not agree with observations which show that for giant galaxies, $\overline{\log M_f}$ is approximately constant and larger than $\overline{\log M_f}$ of clusters in dwarf galaxies.
- adopting a power-law initial GCMF, a significant dependence of $\overline{\log M_f}$ on the galactocentric distance is produced by the effects of evolutionary processes; this is in contrast with several observational studies which fail to find a significant radial gradient of $\overline{\log M_f}$ within individual galaxies.
- Starting with an initial power-law GCMF, values of $\overline{\log M_f}$ consistent with those observed in massive galaxies can be obtained only with values of α smaller ($\alpha \simeq 1.2 - 1.3$) than those observed in young cluster systems.

In Vesperini (2000) it has been shown that the final properties of GCS obtained starting with a log-normal initial GCMF similar to that observed in the external regions of some elliptical galaxies (where evolutionary processes are unlikely to have significantly altered the initial conditions of clusters) are in general good agreement with the observed properties of old cluster systems. In this paper we have shown that, starting with a power-law initial GCMF, the final GCS properties present several discrepancies from those observed in old GCS; this would seem to rule out the possibility that old cluster systems were formed with a power-law initial GCMF similar to that observed in young cluster systems of merging galaxies and to strongly favour a log-normal initial GCMF.

One can not exclude the existence of differences in the process of globular cluster formation at the current epoch and at the time of formation of currently old clusters leading to different initial GCMF. While individual young clusters could indeed be young globular clusters and those which survive could evolve into systems similar to old globular clusters, the future global properties of these young cluster systems could differ from those of currently old cluster systems.

ACKNOWLEDGMENTS

I wish to thank an anonymous referee for useful comments on the paper. Support from a Five College Astronomy Department fellowship is acknowledged.

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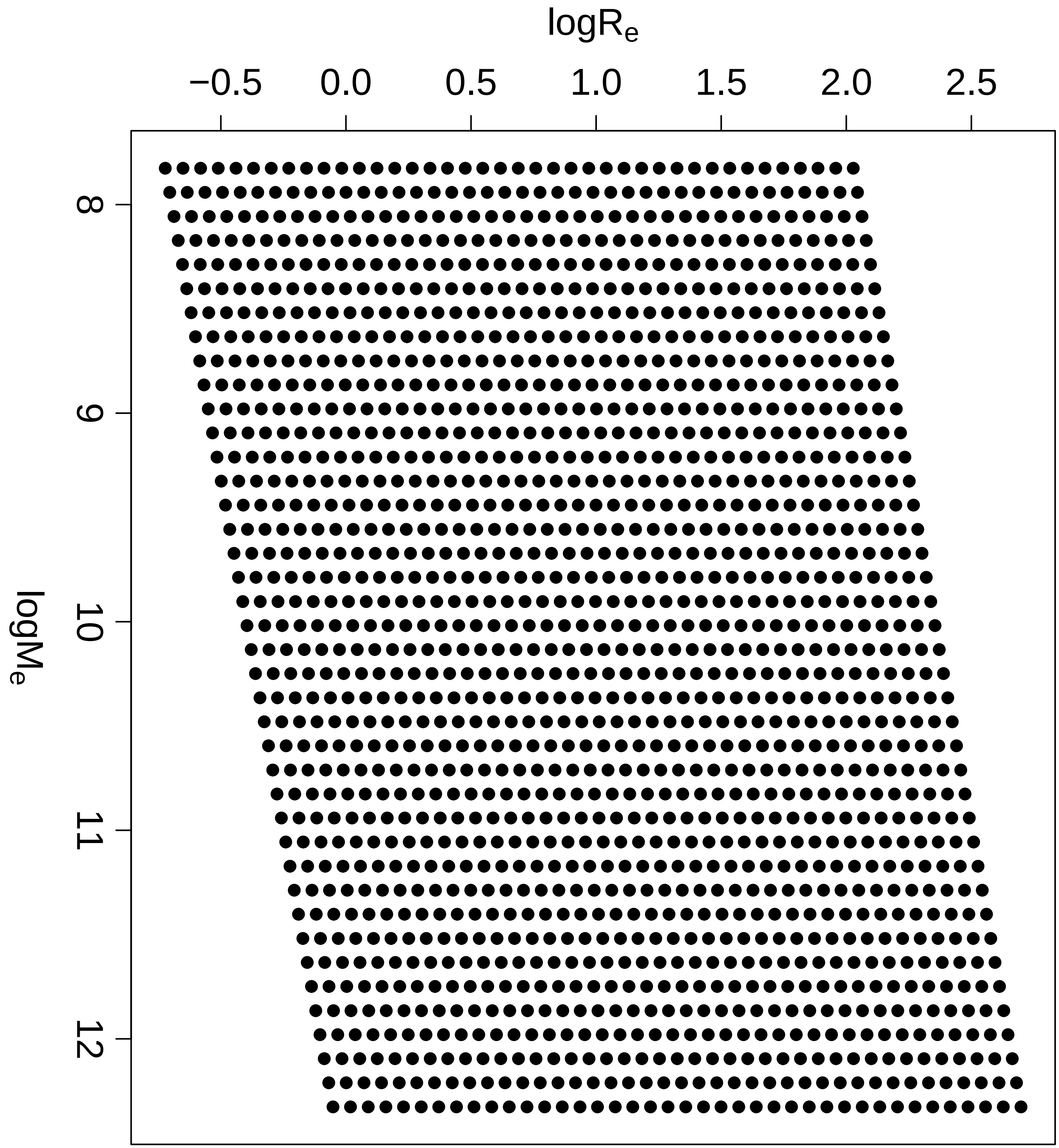
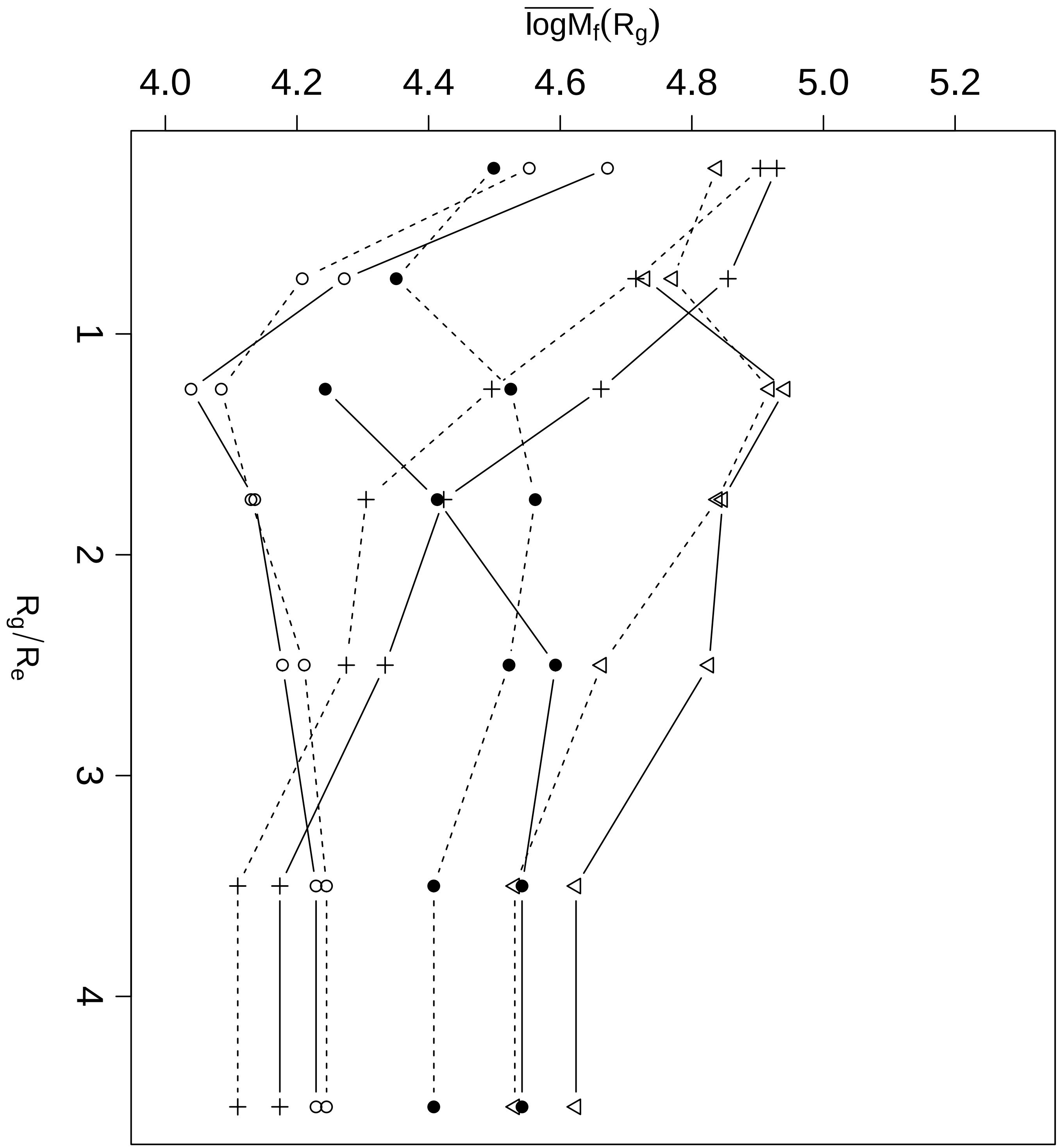
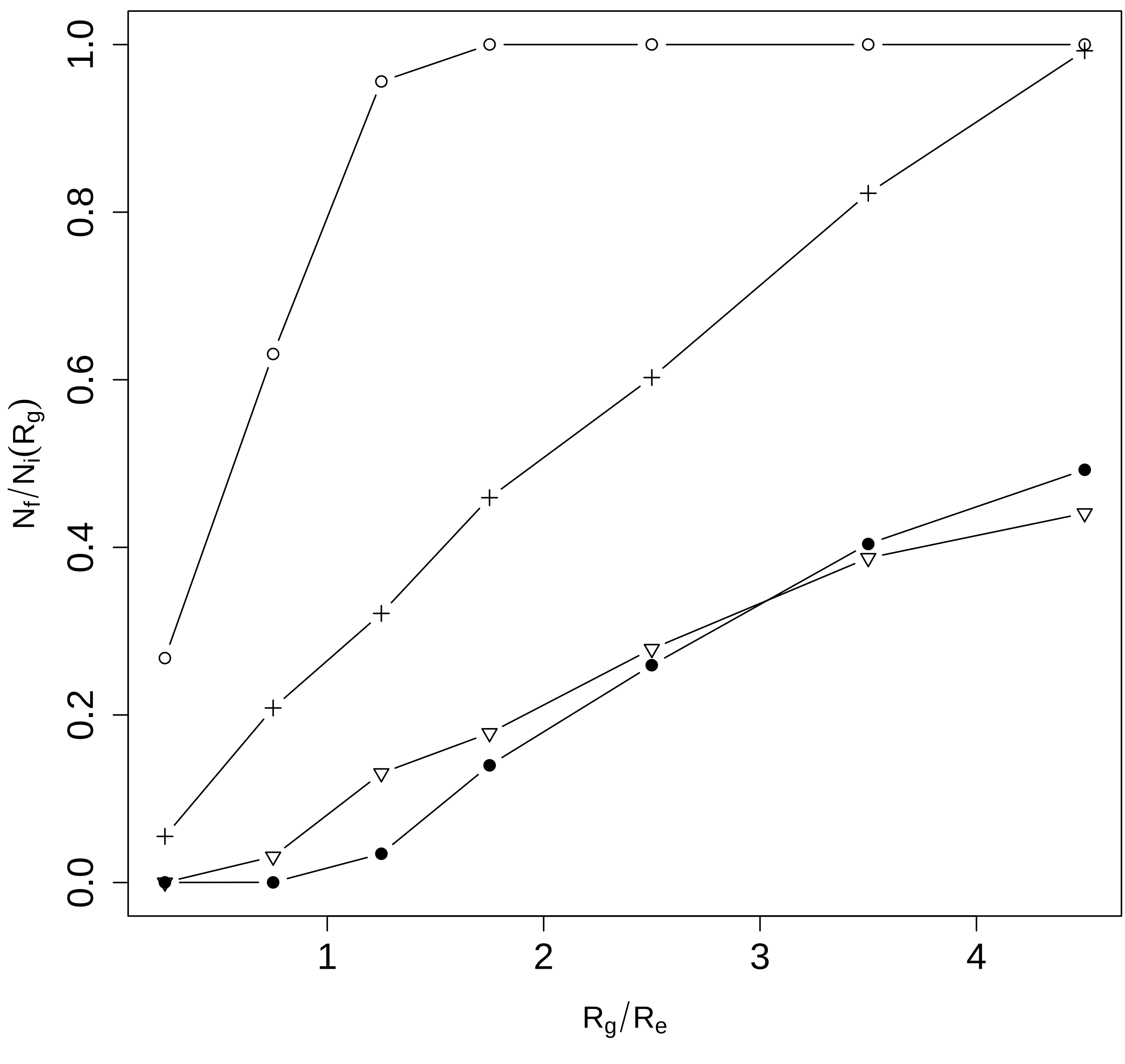
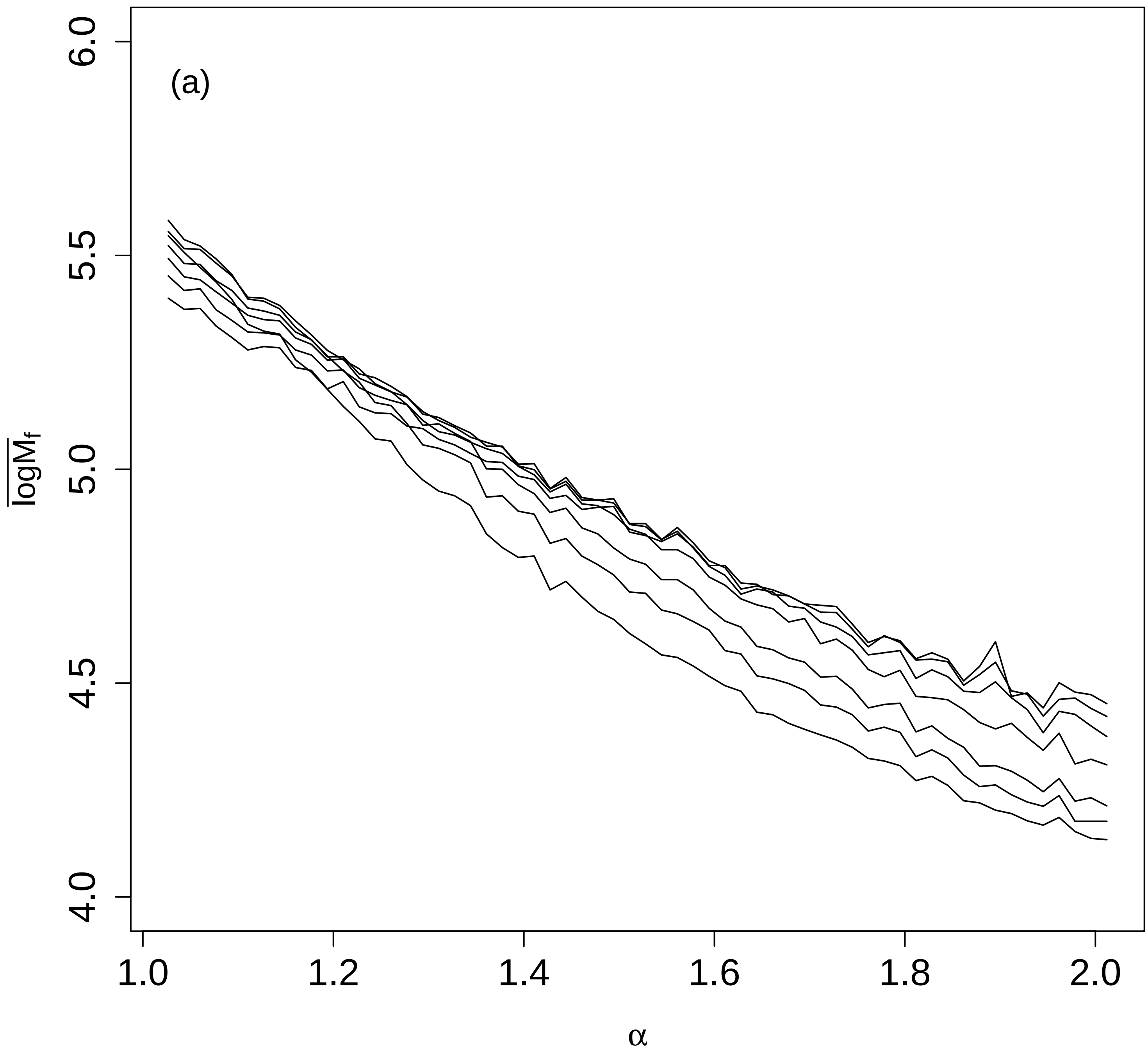


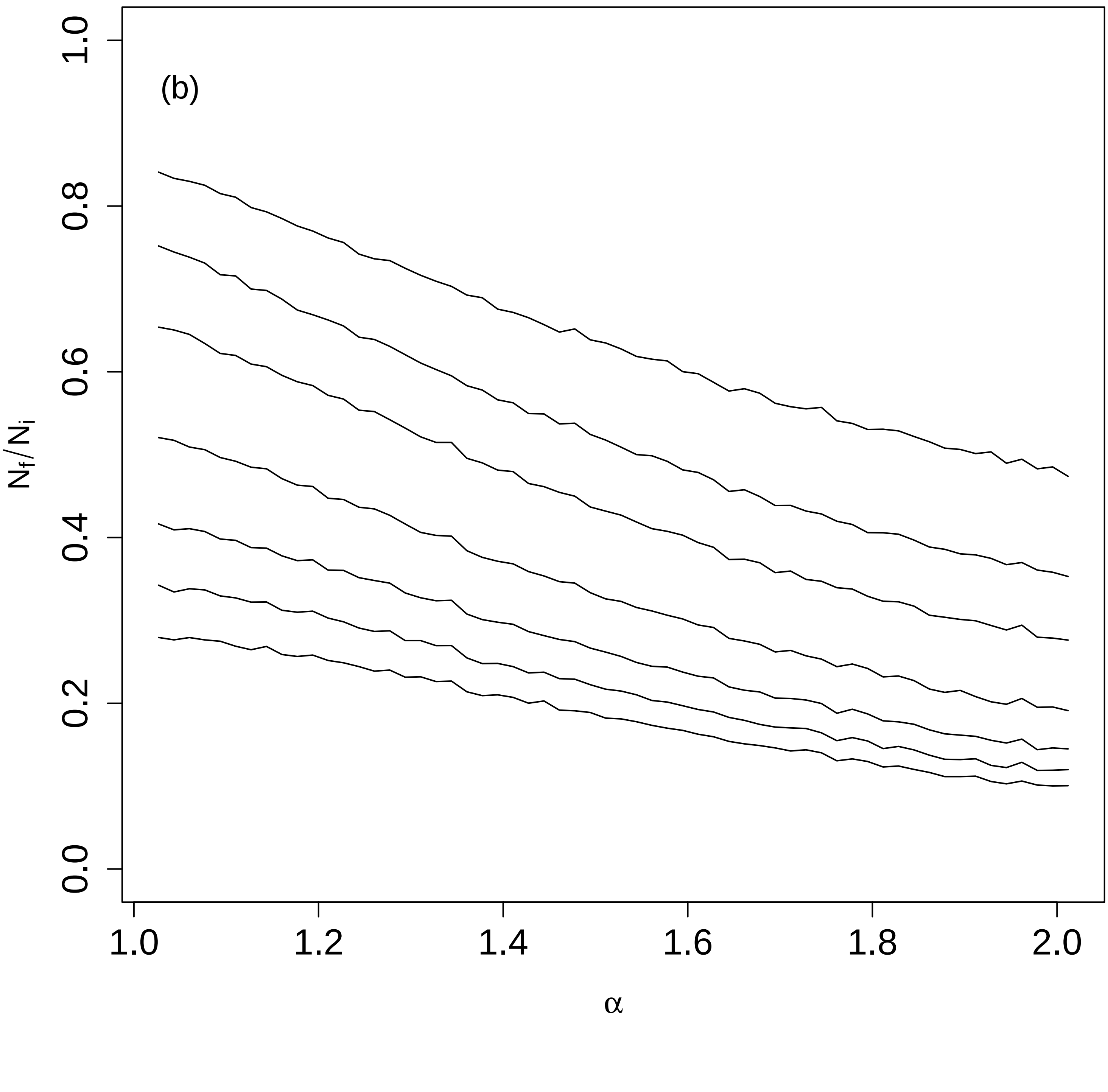
logRe
−0.5
0.0
0.5
1.0
1.5
2.0
2.5
logMe
8
9
10
11
12



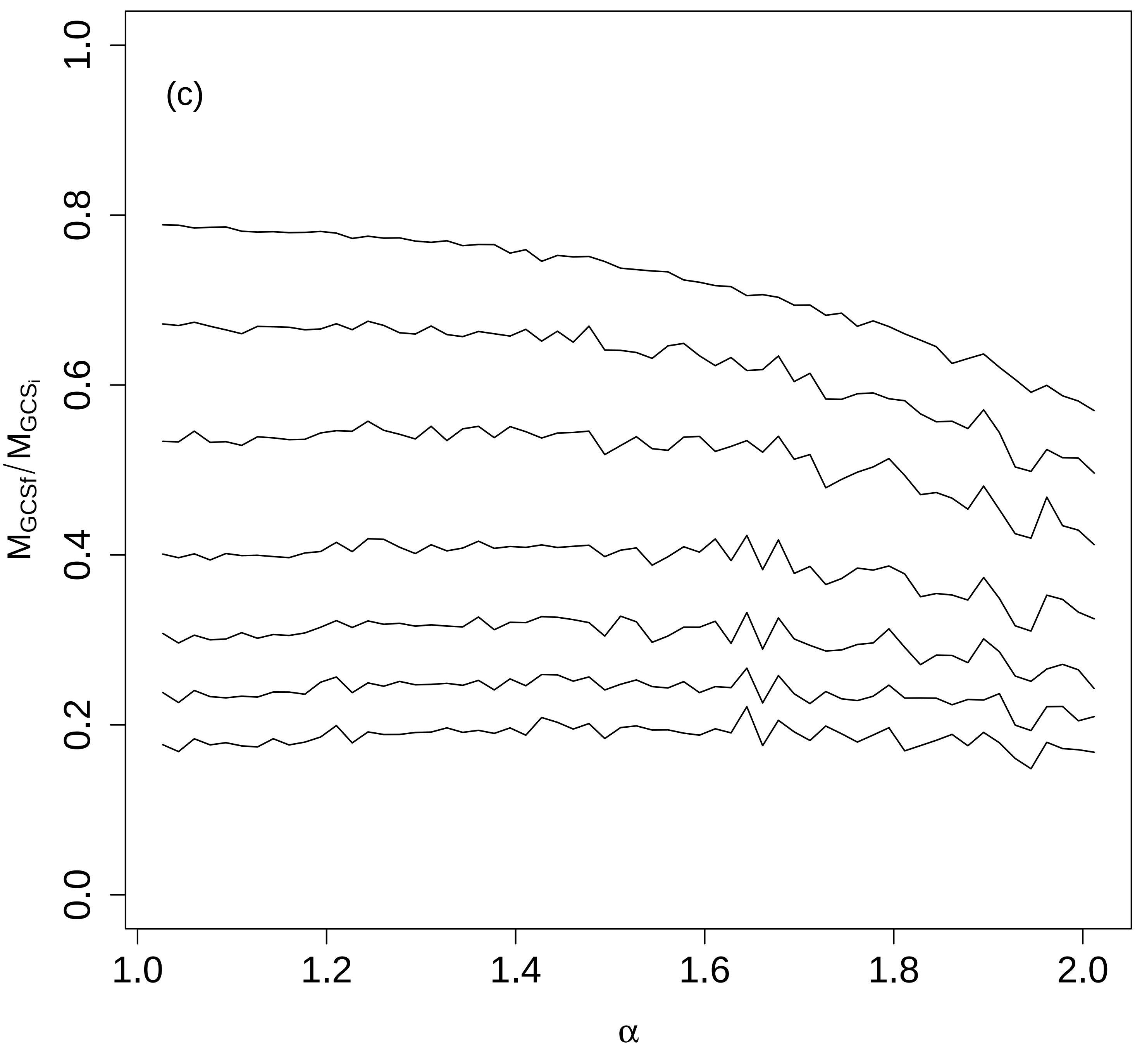
$\overline{\log M_f}(R_g)$
4.0
4.2
4.4
4.6
4.8
5.0
5.2
R_g/R_e
1
2
3
4


$N_f/N_i(R_g)$
0.0
0.2
0.4
0.6
0.8
1.0
R_g/R_e
1
2
3
4

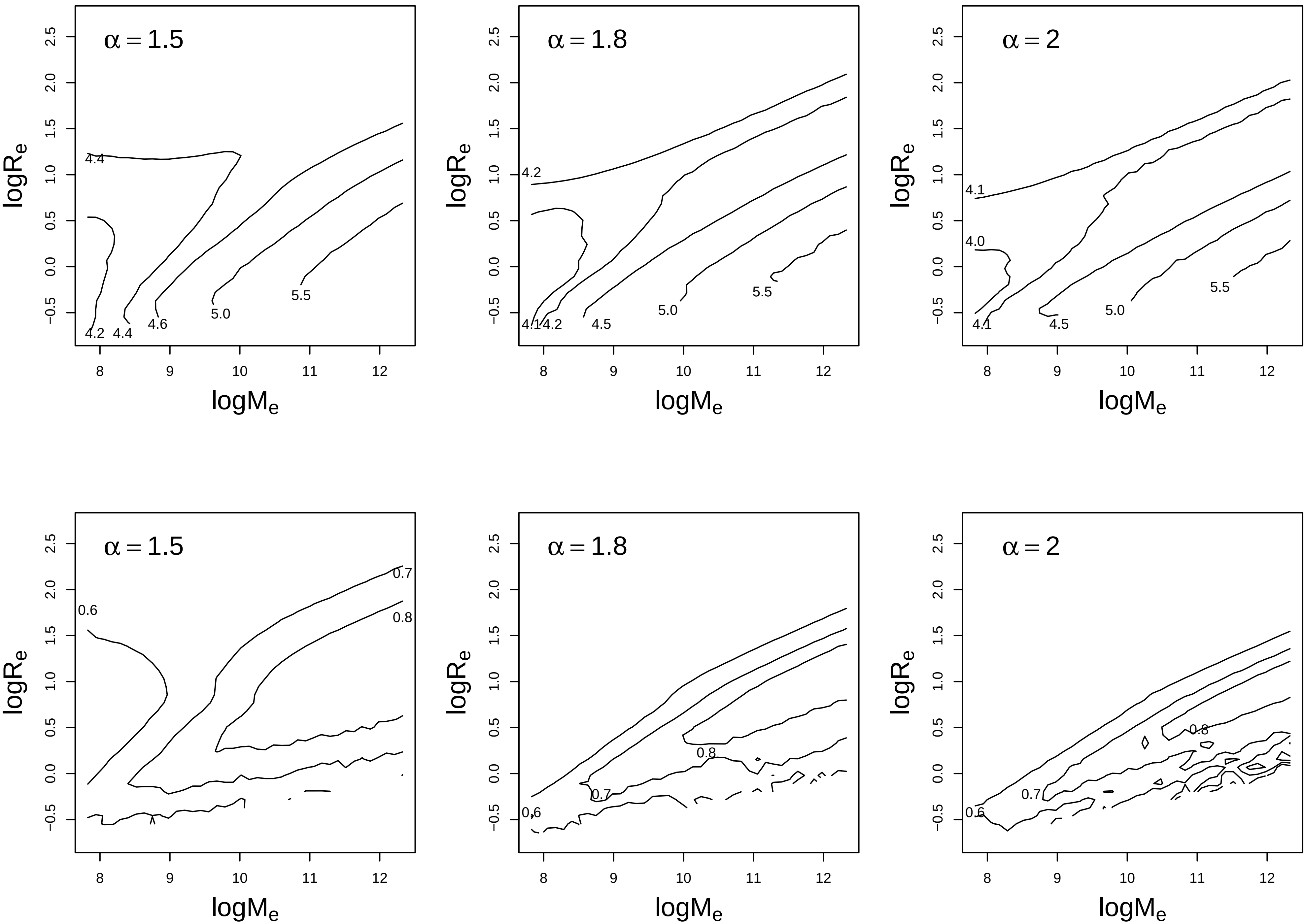

(a)
6.0
5.5
5.0
4.5
4.0
1.0
1.2
1.4
1.6
1.8
2.0
α
$\overline{\log M_f}$



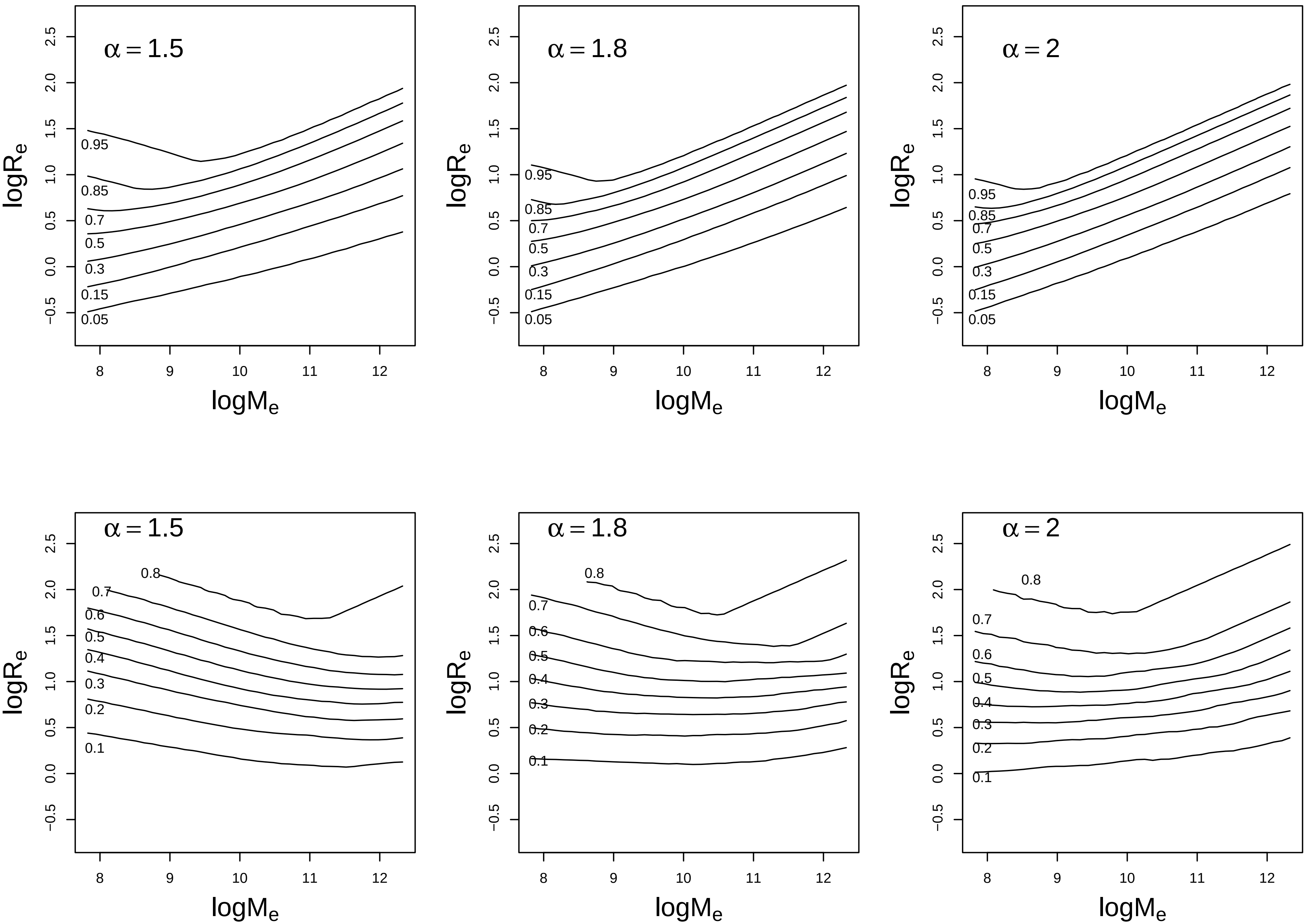
(b)
1.0
0.8
0.6
0.4
0.2
0.0
N_f/N_i
1.0
1.2
1.4
1.6
1.8
2.0
α



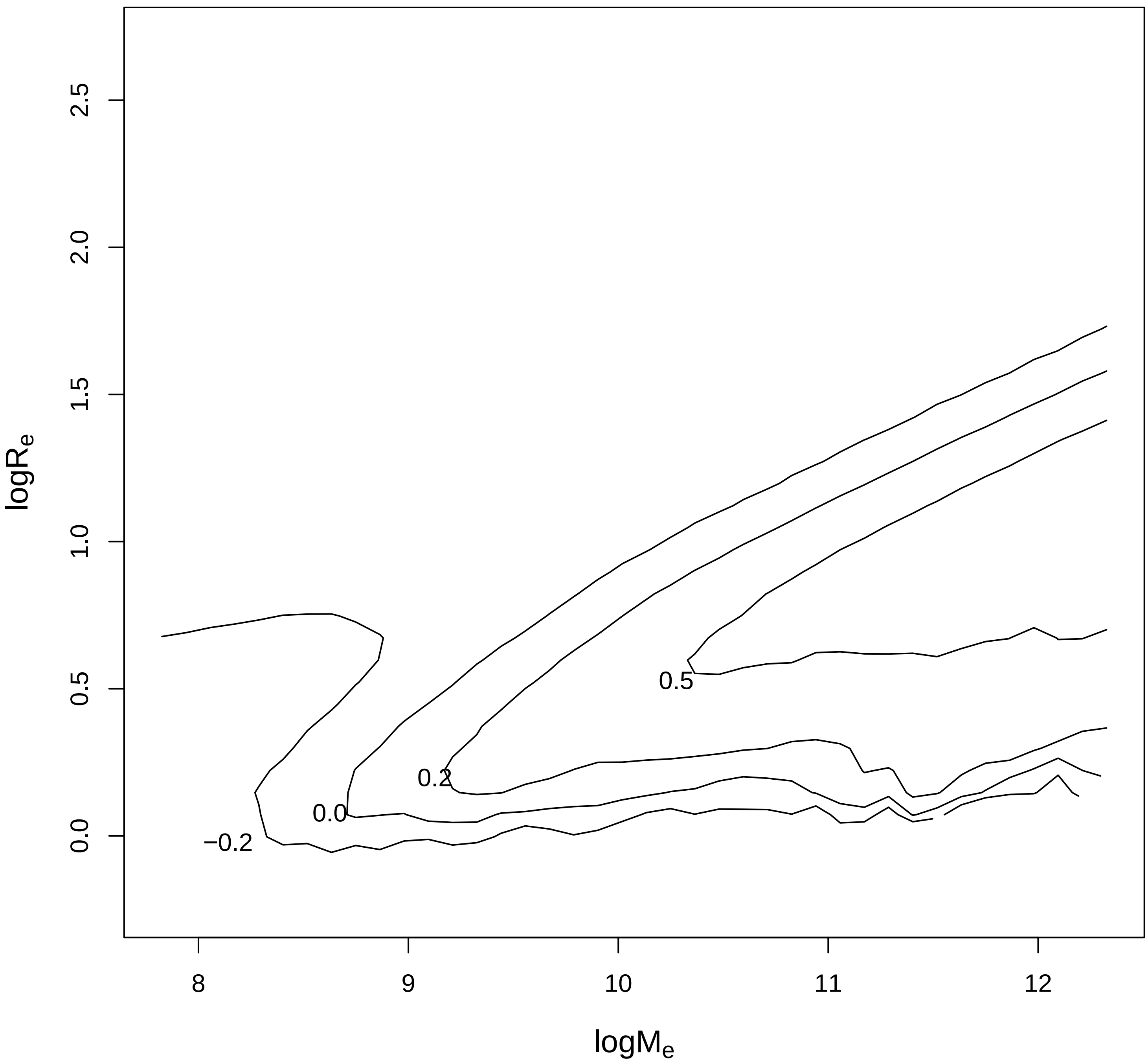
(c)
M_{GCSf}/M_{GCS_i}
0.0
0.2
0.4
0.6
0.8
1.0
1.0
1.2
1.4
1.6
1.8
2.0
α



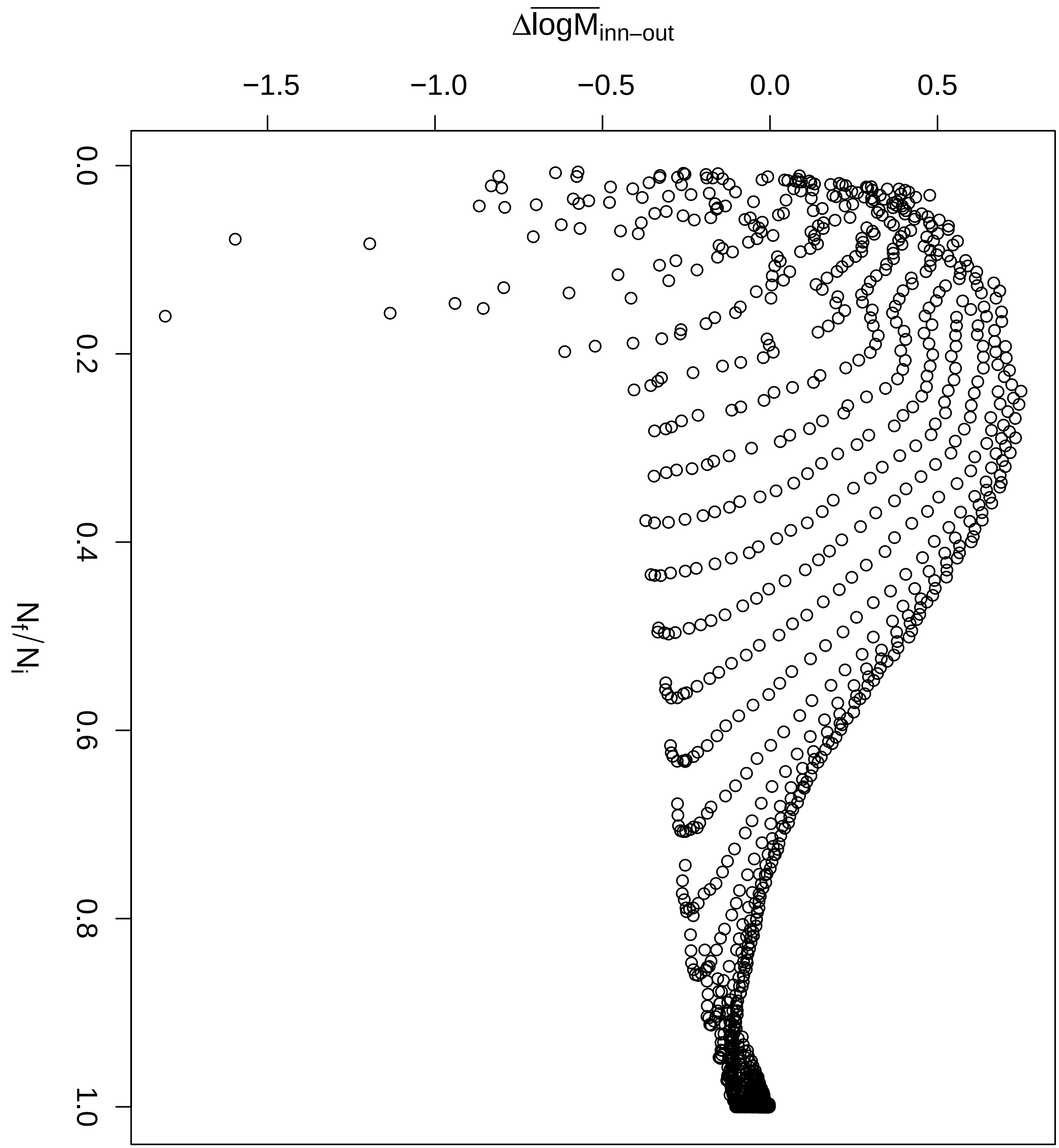
α = 1.5
α = 1.8
α = 2
logRe
logMe
4.4
4.2
4.4
4.6
5.0
5.5
4.2
4.1
4.2
4.5
5.0
5.5
4.1
4.0
4.1
4.5
5.0
5.5
0.6
0.7
0.8
0.6
0.7
0.8
0.6
0.7
0.8

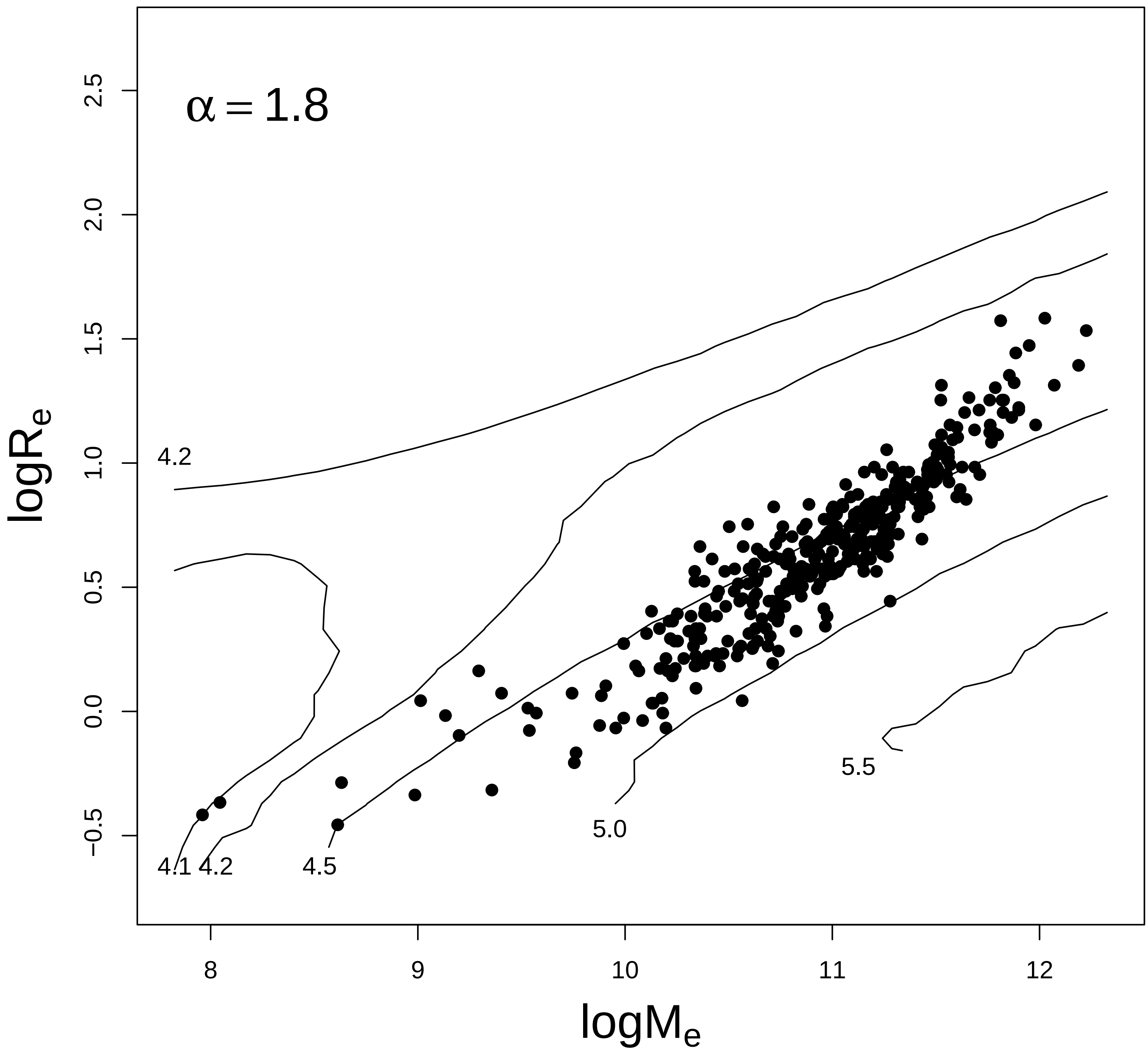


α = 1.5
α = 1.8
α = 2
logR_e
logM_e
0.95
0.85
0.7
0.5
0.3
0.15
0.05
α = 1.5
α = 1.8
α = 2
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1

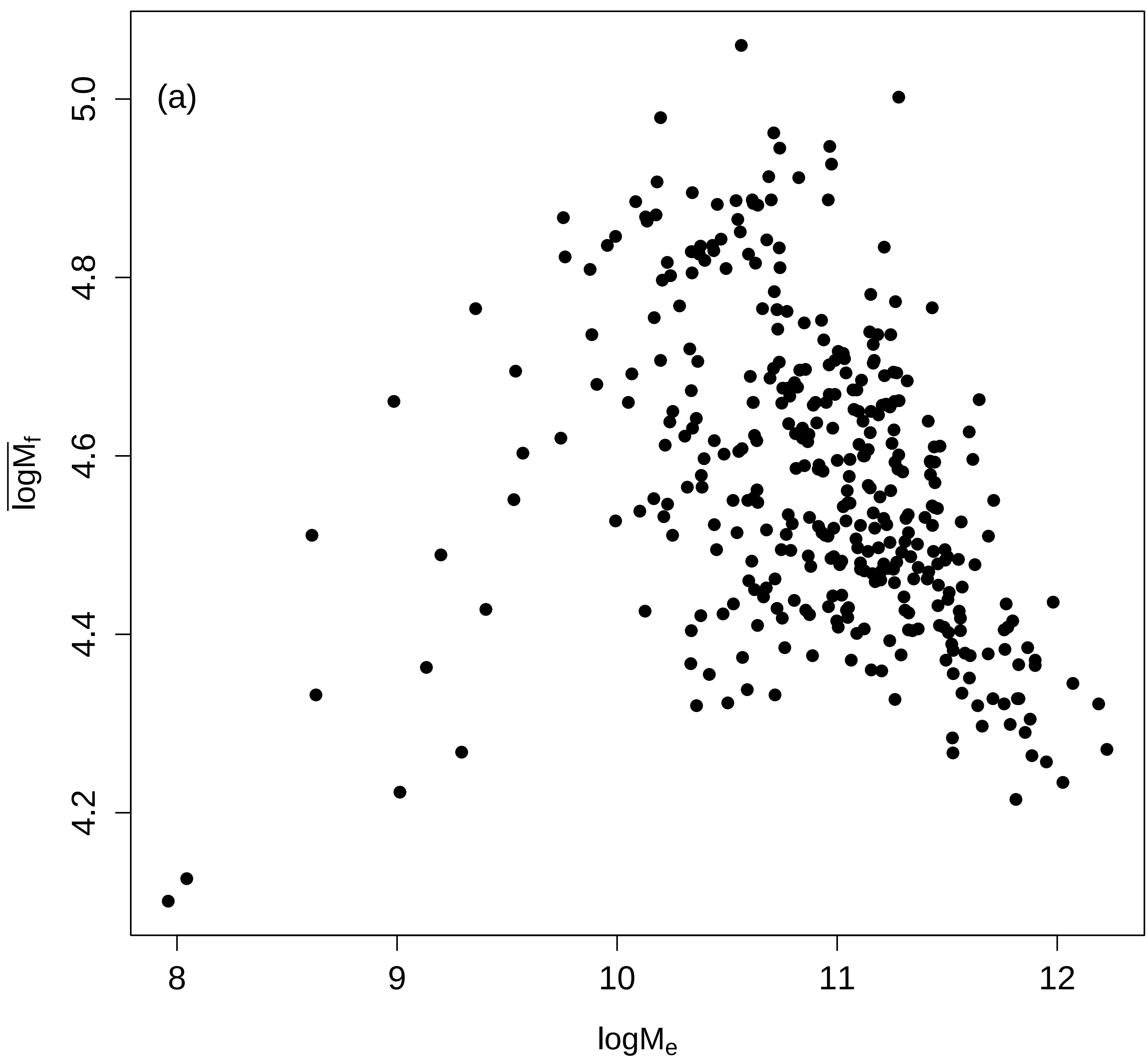


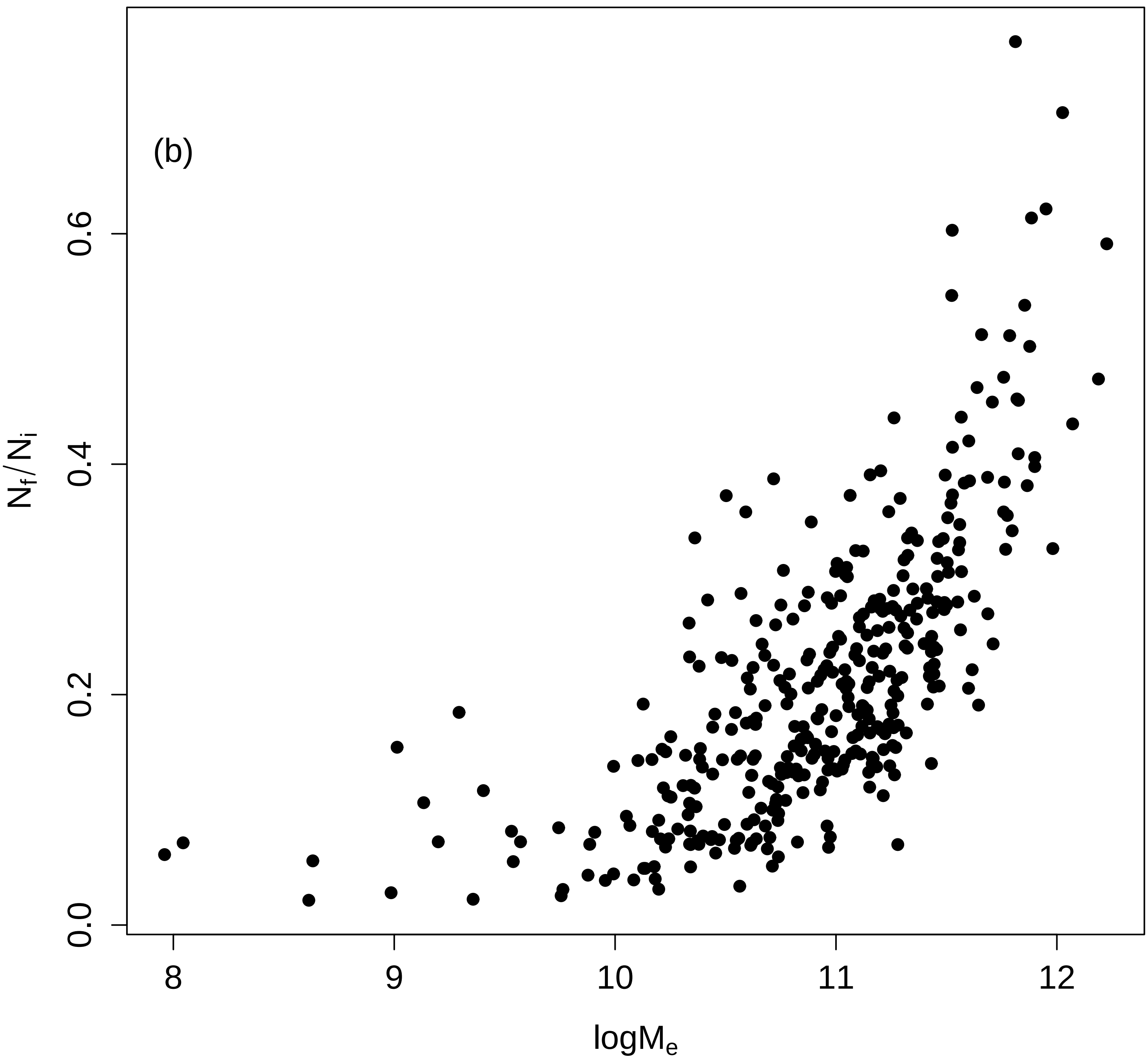
2.5
2.0
1.5
1.0
0.5
0.0
logR_e
8
9
10
11
12
logM_e
0.5
0.2
0.0
−0.2

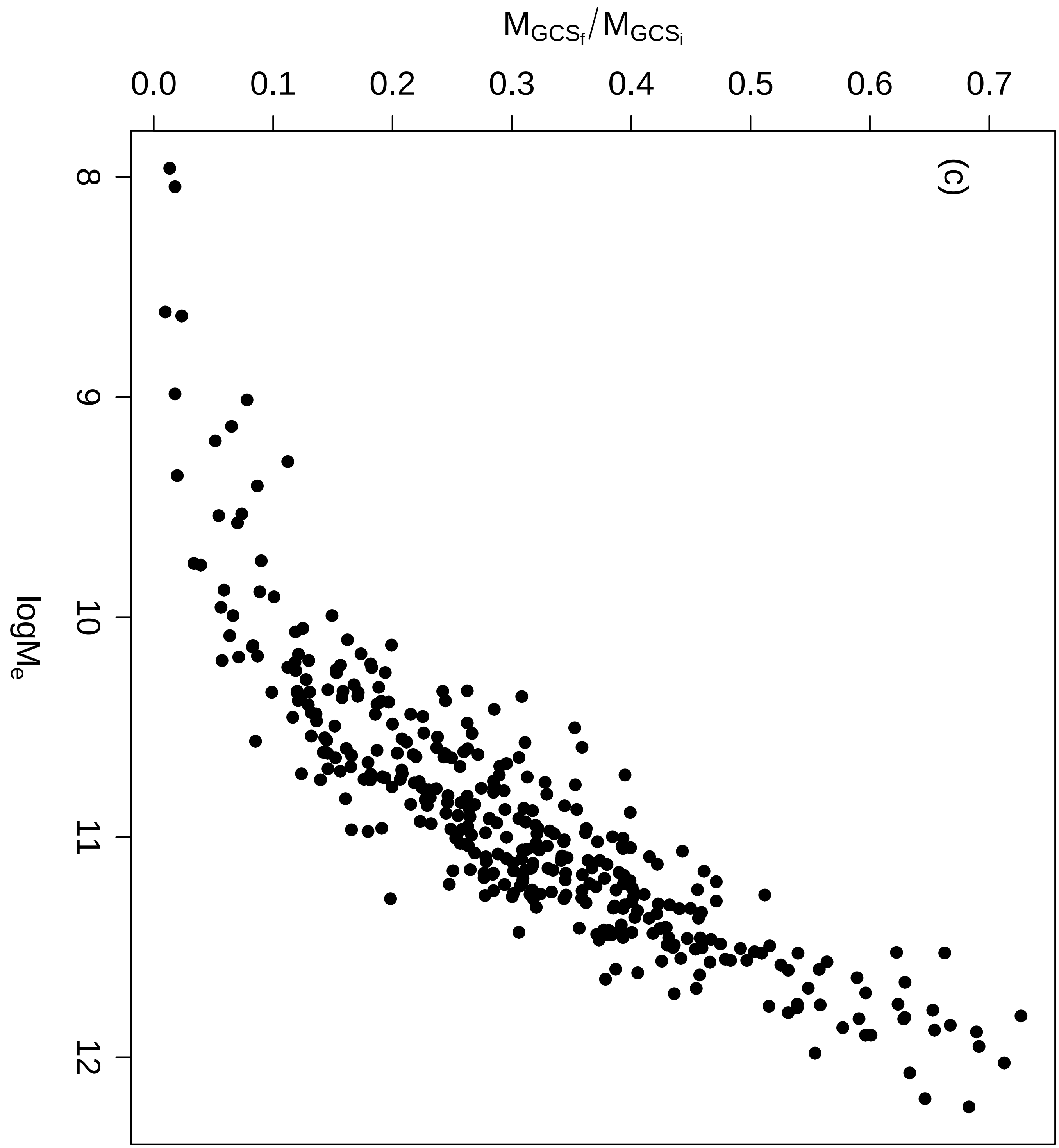

$\Delta\overline{\log M}_{inn-out}$
−1.5
−1.0
−0.5
0.0
0.5
N_f/N_i
0.0
0.2
0.4
0.6
0.8
1.0



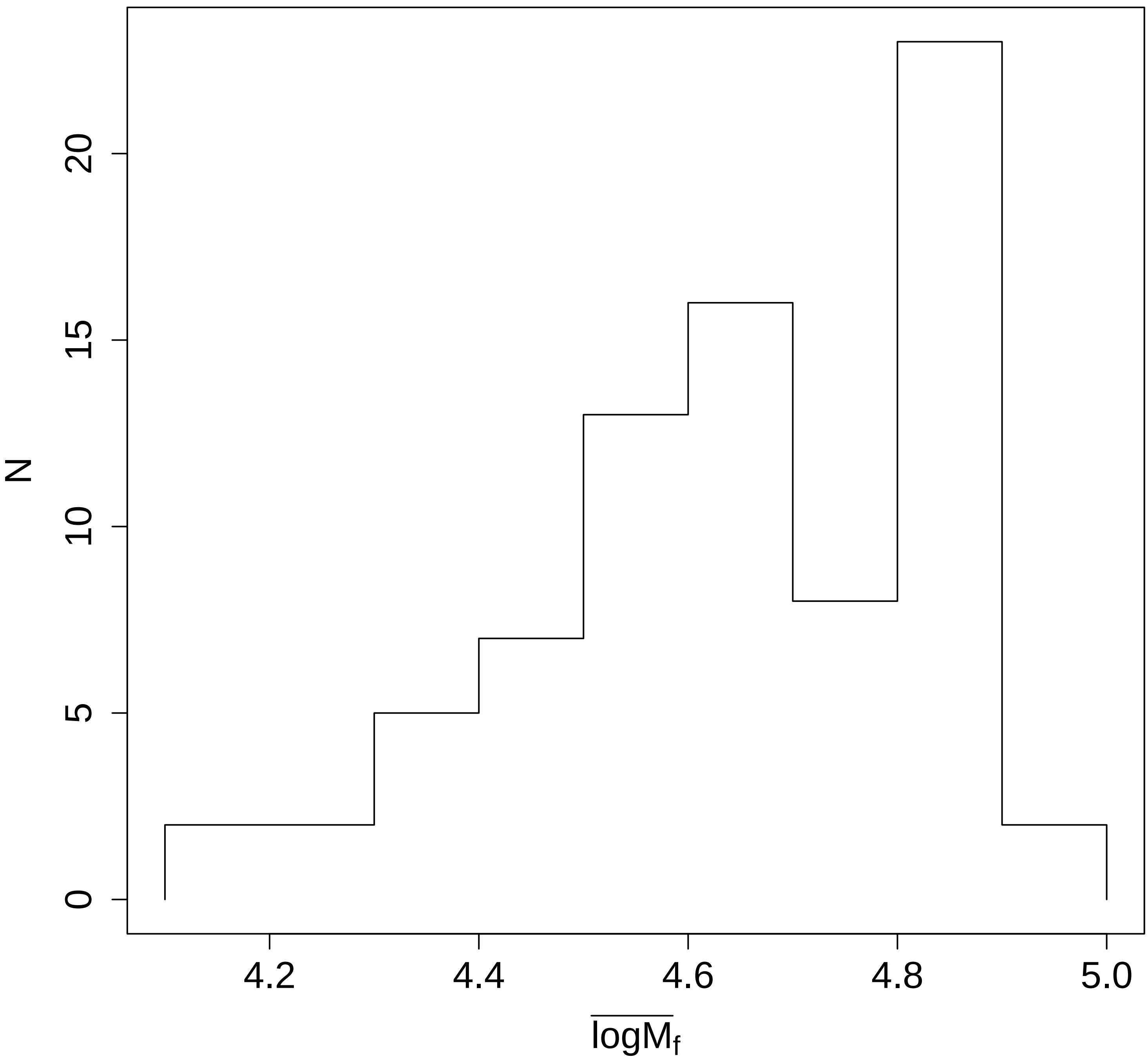
α = 1.8
logR_e
logM_e
4.2
4.1
4.2
4.5
5.0
5.5
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
8
9
10
11
12


(a)
5.0
4.8
4.6
4.4
4.2
$\overline{\log M_f}$
8
9
10
11
12
$\log M_e$

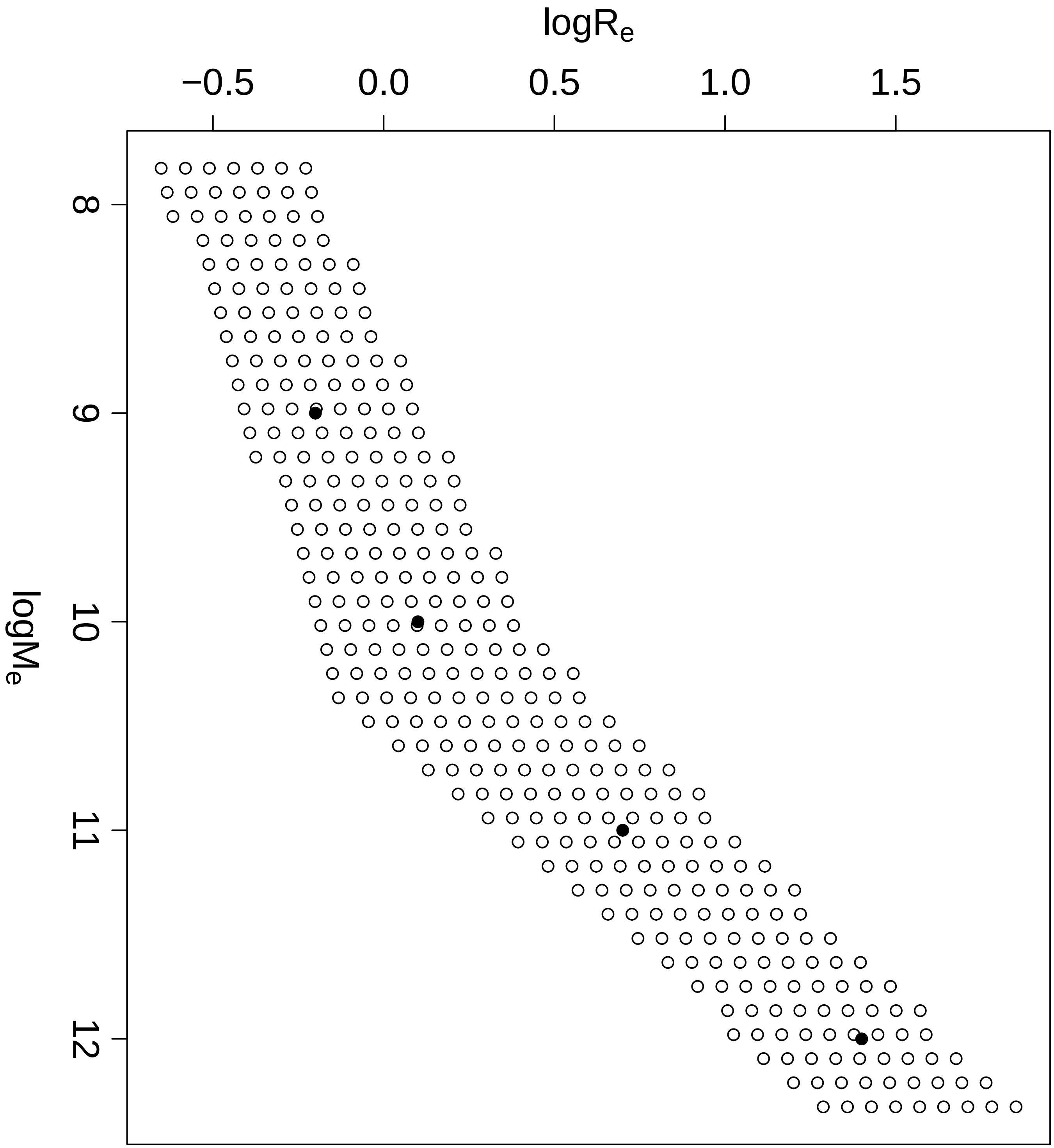

(b)
N_f/N_i
logM$_e$
0.0
0.2
0.4
0.6
8
9
10
11
12

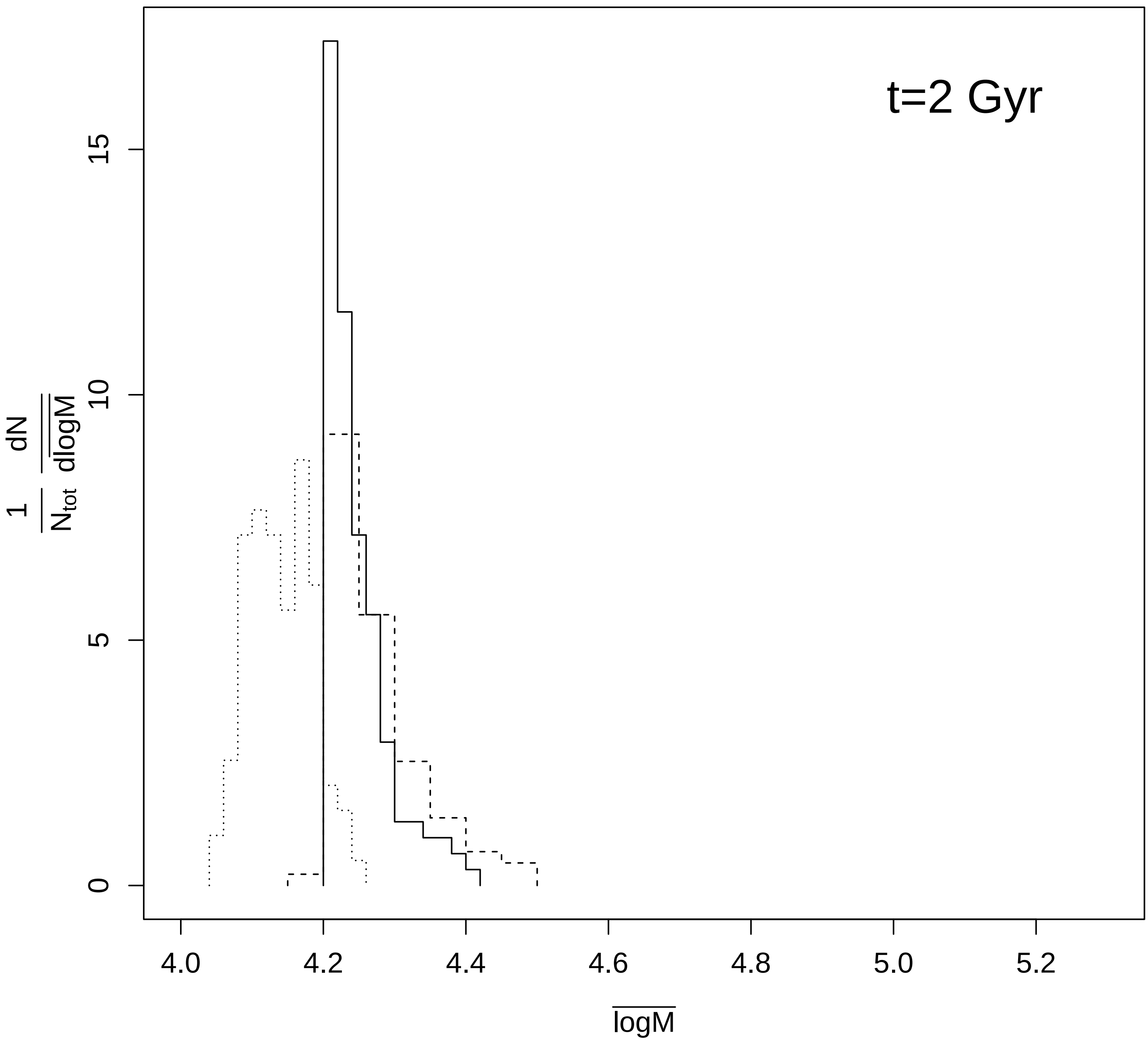


(c)
M_GCSf/M_GCSi
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
logMe
8
9
10
11
12

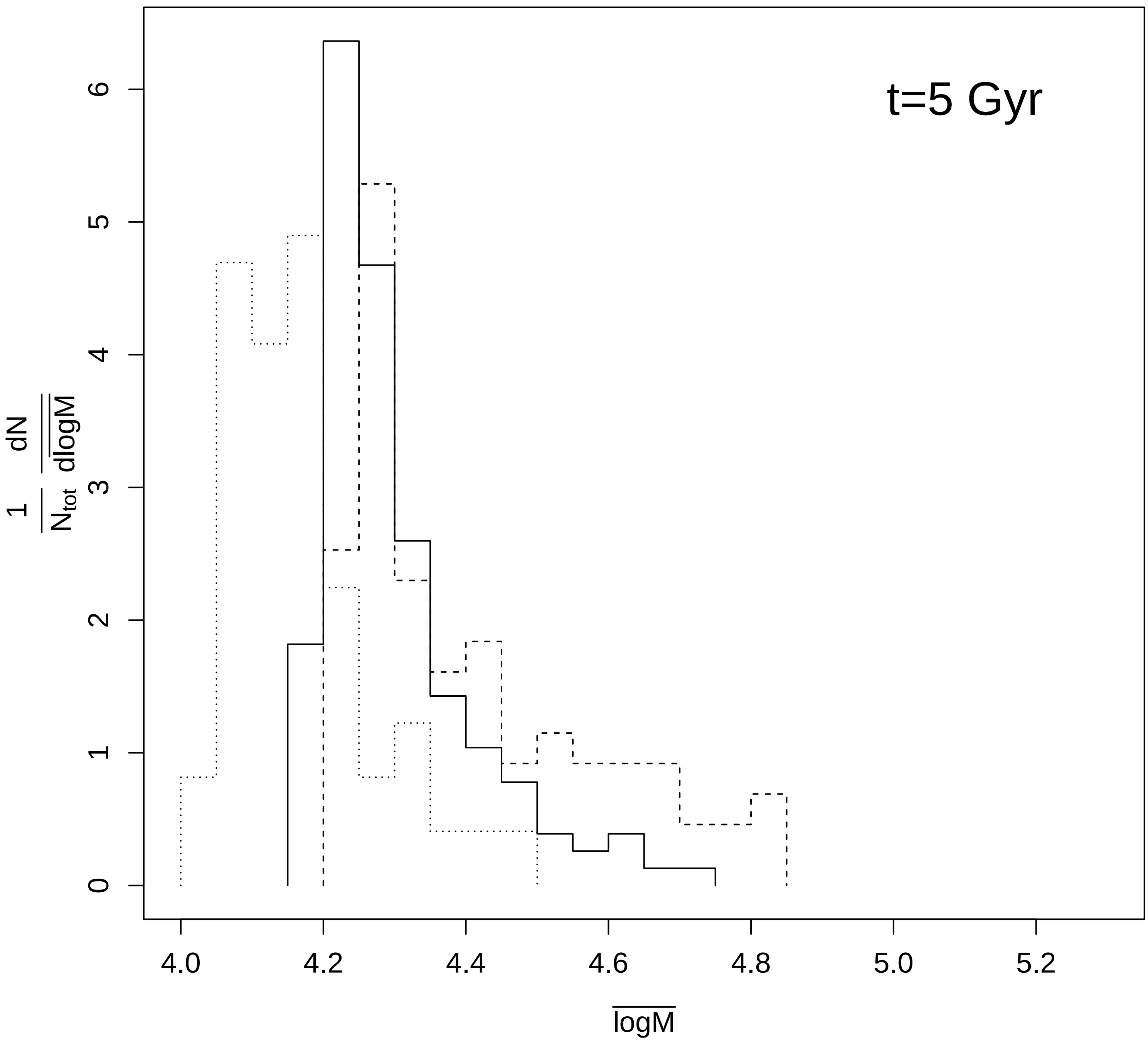


N
0
5
10
15
20
4.2
4.4
4.6
4.8
5.0
$\overline{\log M_f}$

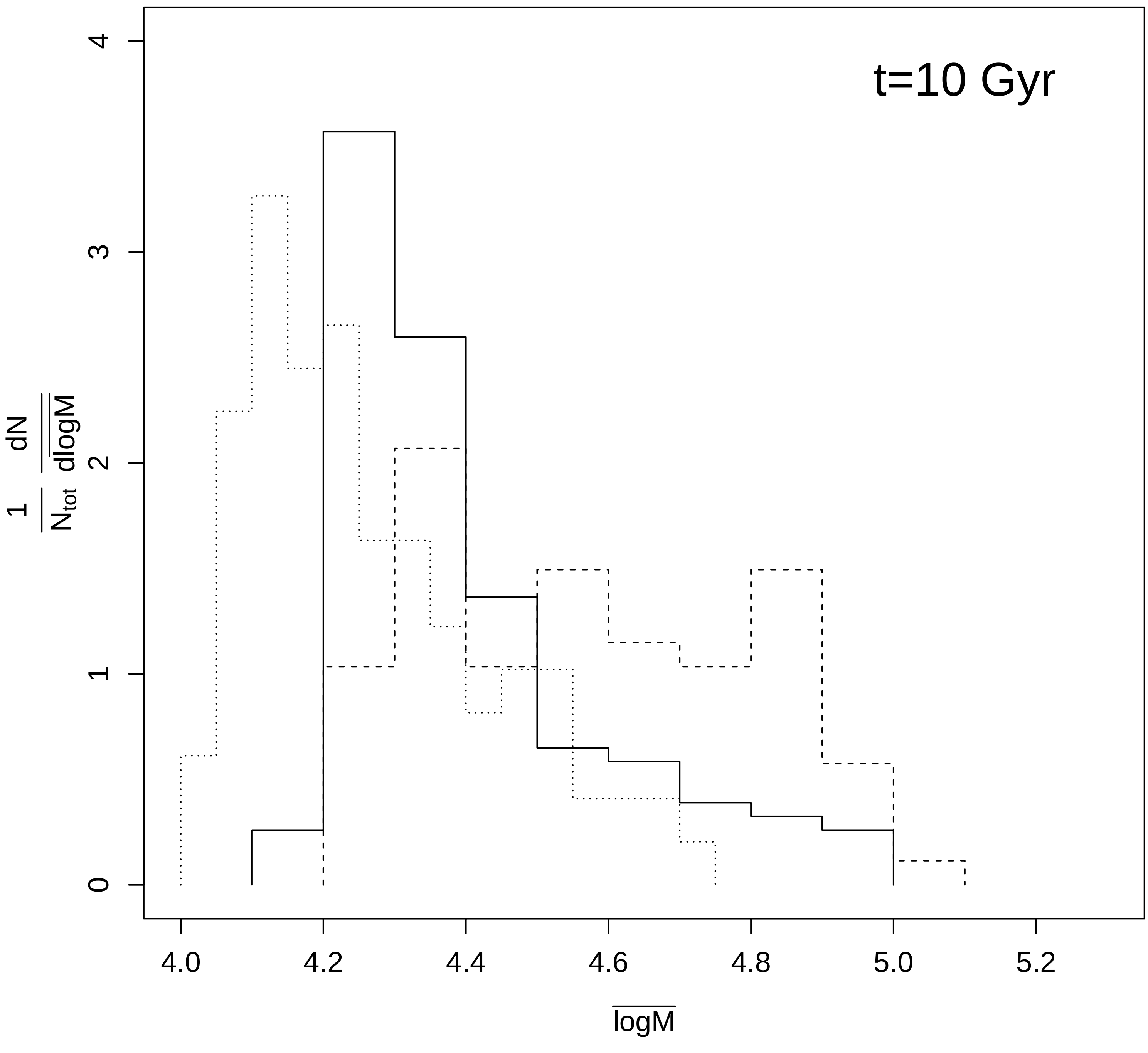

logR$_e$
−0.5
0.0
0.5
1.0
1.5
logM$_e$
8
9
10
11
12

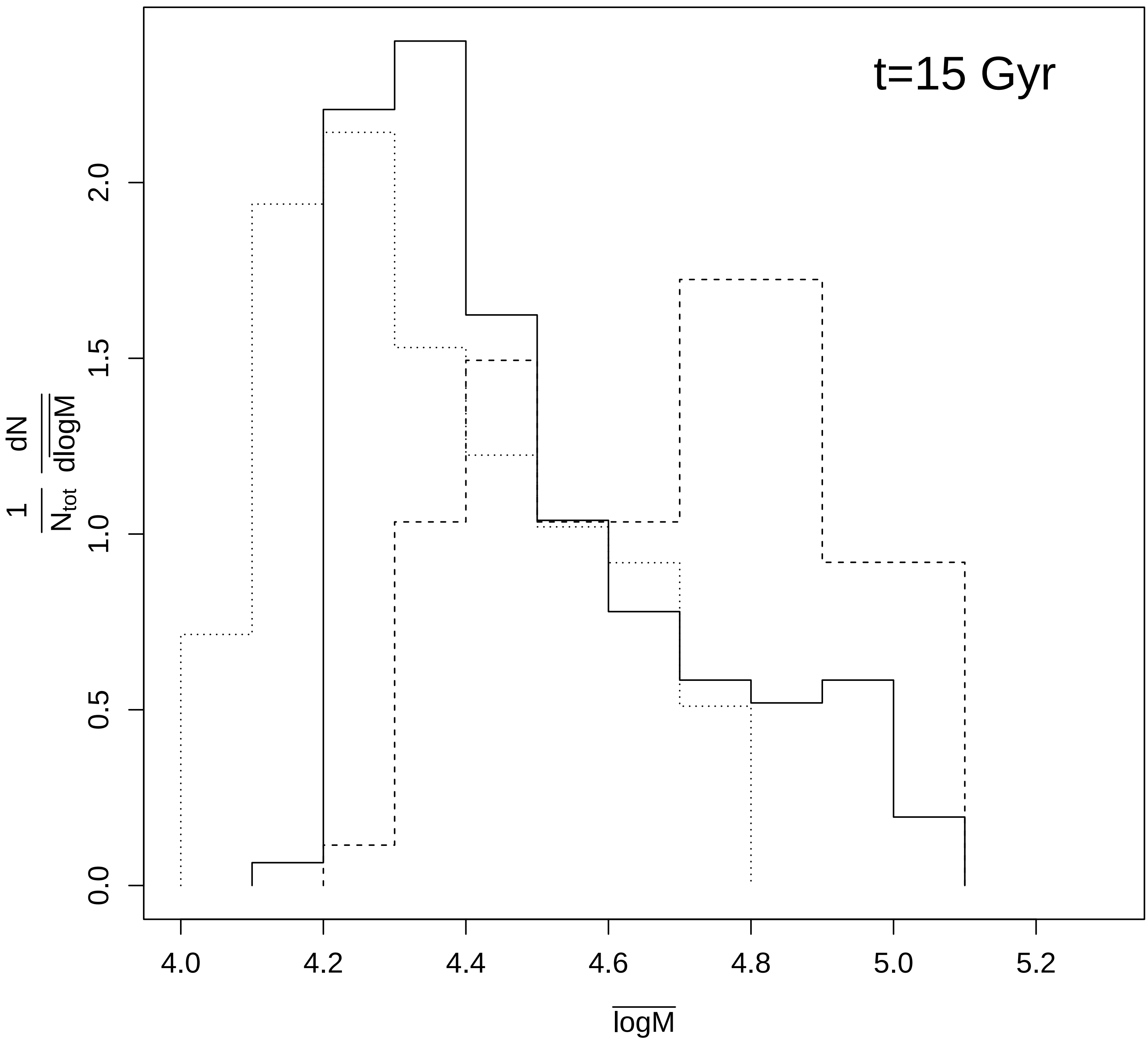


t=2 Gyr
$\frac{1}{N_{tot}} \frac{dN}{d\overline{\log M}}$
0
5
10
15
4.0
4.2
4.4
4.6
4.8
5.0
5.2
$\overline{\log M}$



t=5 Gyr
0
1
2
3
4
5
6
$\frac{1}{N_{tot}} \frac{dN}{d\overline{\log M}}$
4.0
4.2
4.4
4.6
4.8
5.0
5.2
$\overline{\log M}$


t=10 Gyr
0
1
2
3
4
4.0
4.2
4.4
4.6
4.8
5.0
5.2
logM
1/N_tot dN/dlogM


t=15 Gyr
2.0
1.5
1.0
0.5
0.0
4.0
4.2
4.4
4.6
4.8
5.0
5.2
$\frac{1}{N_{tot}} \frac{dN}{d\overline{\log M}}$
$\overline{\log M}$